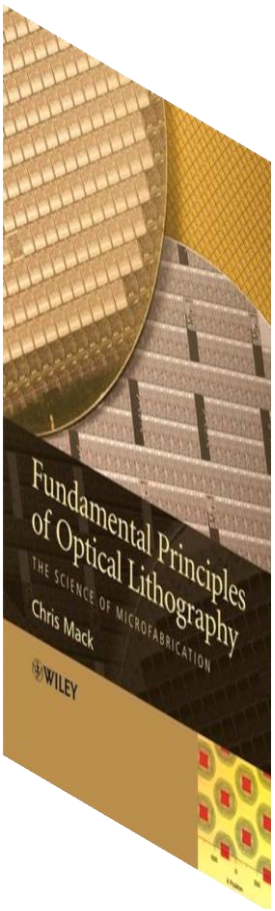
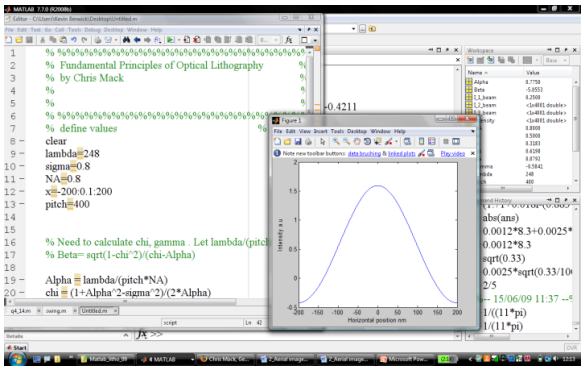
Laboratory Manual to accompany Fundamental Principles of Optical Lithography, by Chris Mack

2011

Optical Lithography Modelling with MATLAB®





Kevin Berwick

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Foreword

Like virtually any area of study, optical lithography is best learned by a combination of thinking and doing. Reading my textbook, *Fundamental Principles of Optical Lithography,* will certainly require a fair amount of thinking. And doing the homework problems is a first step in "doing", practicing lithography to reinforce the concepts found in the book. But the real practice of lithography – getting in the fab or lab and printing wafers – is often out of the reach of most students, requiring equipment and facilities that are frequently unavailable. Thus, there is a need for more opportunities to "do" than the book's homework problems can provide. Simulating lithography, performing the complex calculations detailed in my book to create a virtual lithography environment, is a valuable way to get experiences closer to the real world of lithography practice without the (sometimes extreme) expense of physical experiments.

While sophisticated lithography simulators such as PROLITH® are extremely valuable learning tools for the student of lithography, Kevin Berwick in this volume offers another approach to learning through simulation: use the power of the MATLAB® environment to enable students to write their own code for performing complex lithography calculations. Found within these pages are laboratory "experiments" to be performed in the MATLAB® environment. They are, in a sense, extreme homework problems, requiring computational capabilities that often stretch far beyond paper, pencil and calculator. But their very complexity enables a new level of learning – the ability to toy with inputs and examine the resulting outputs in a simulated lithography experience. While seeing a graph showing how an output varies with an input can be instructive, doing the calculations that generate that curve is even more instructive. Kevin Berwick's MATLAB® laboratory manual is a valuable resource for the reader of *Fundamental Principles of Optical Lithography*. Whether in academia or in industry, performing the laboratories found in this manual will bring one's lithography learning a little closer to reality.

Chris Mack
December, 2010
Austin, Texas, USA

Preface

The original motivation to write this book was when I was asked to teach an elective, one-semester course in Lithography as part of a new M. Sc. Program at the Dublin Institute of Technology. While there are many books available which describe the full range of processes required to build a circuit on silicon, very few exist which concentrate solely on Lithography.

One of the aspects of Lithography which I wanted to highlight was mathematical modelling of Lithography and, as I searched the literature in order to find information on this for my lecture notes, Chris Mack's name cropped up again and again. As the author of the lithography simulation software *PROLITH*®, Chris has an enormous breadth of knowledge on the subject of lithography modelling. I was delighted when, in 2007, Chris's book, *Fundamental Principles of Optical Lithography*, was published. Here was a synthesis of the many, many papers published in the field, filtered by an expert and dished up in an easily digestible form for my reading pleasure.

For about a month, my morning routine before starting my teaching consisted of a train journey into town, followed by two double shot lattes at *Insomnia*, my favourite coffee shop, as I read Chris's book from cover to cover. The mathematics was, in parts, intimidating. Although largely standard algebra and calculus, many of the expressions developed were long. In addition, the interpretation of the mathematics was challenging, requiring me to dust off some of the physics I hadn't used for the best part of a decade. As an engineer, I found that the plots of the equations in the text were a great help in visualising the underlying physics.

Initially, I wrongly assumed that all the plots had been generated using *PROLITH*®. However, as I read further, it became apparent that most of the Figures could be reproduced with any mathematical calculation and plotting program.

Coincidentally, I had started to use MATLAB® for teaching several other subjects around this time. MATLAB® allows you to develop mathematical models quickly, using powerful language constructs, and is used in almost every Engineering School on Earth. MATLAB® has a particular strength in data visualisation, allowing you to get insight into trends in data far quicker than using spreadsheets or other high level languages such as C or Fortran. I decided to attempt to reproduce some of the plots in Chris's book using MATLAB® and still remember my delight when, after writing only a few lines of MATLAB® code, an Aerial Image plot identical to that in the book materialised on my screen.

In my experience, familiarity with theoretical models, and the underlying physical principles used in constructing the models, is greatly improved if the behaviour of the equations is explored using computer software. So, I wrote several laboratory exercises for my students based on Chris's book using MATLAB®. The lab sheets I produced eventually grew into the book you are now reading.

This book is intended to act as a companion to Chris's book, reinforcing the principles covered in the text. As such, I would imagine the Manual will prove useful to Senior Undergraduate and Graduate students, as well as practising lithographers who wish to improve their theoretical knowledge of what is essentially a practical process. Along the way, the interested student will gain some familiarity with MATLAB®. In order to kick start this process, I include a short MATLAB® Primer as an Appendix to the text.

An Instructor's Solution manual is available by emailing me at kevin.berwick@dit.ie . Suggestions for improvements, error reports and additions to the book are always welcome and can be sent to me at this email address.

I would like to thank a number of people who assisted in the production of this manual. I have already mentioned Chris Mack, author of *Fundamental Principles of Optical Lithography*. Most of the questions in this Manual are based directly on his textbook. Chris advised me on the mathematics and physics when needed, and supplied me with analytical solutions to many of the problems. He also provided valuable advice at various stages of the production of the book and his enthusiasm for the project when initially proposed was very encouraging.

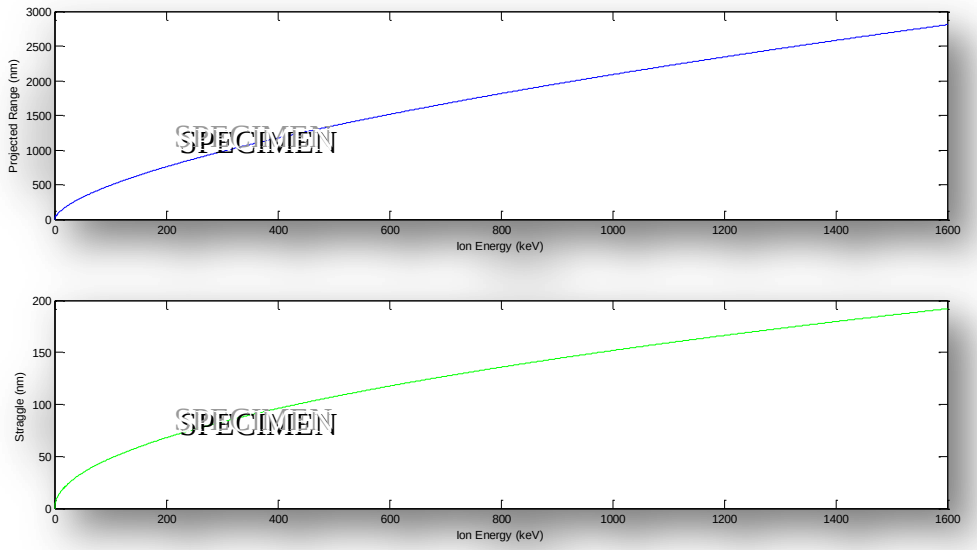
I also received valuable feedback from anonymous reviewers during the writing of the book and am very grateful for their contributions. I would like to thank The MathWorks for their support via their Book program and for producing the MATLAB® software on which this book is based. I would also like to thank the School of Electronics and Communications Engineering at the Dublin Institute of Technology for affording me the opportunity to write this book.

Finally, I would like to thank my family, who tolerated my absence on the many occasions when, largely self imposed, deadlines loomed.

Kevin Berwick
Dublin, Ireland
December 2010

1. Introduction to Semiconductor Lithography

1. Plot the projected range and straggle for the ion implantation of Boron into resist. Use the data in Table 1.1 of the Book. You should get something like this.



(i) What energy is required to produce a Boron implant with a projected range of 1μm?
(ii) What is the straggle at this Energy?

2. The Yield, Production Cost and Profit, in dollars, of a chip at a semiconductor plant can be described by

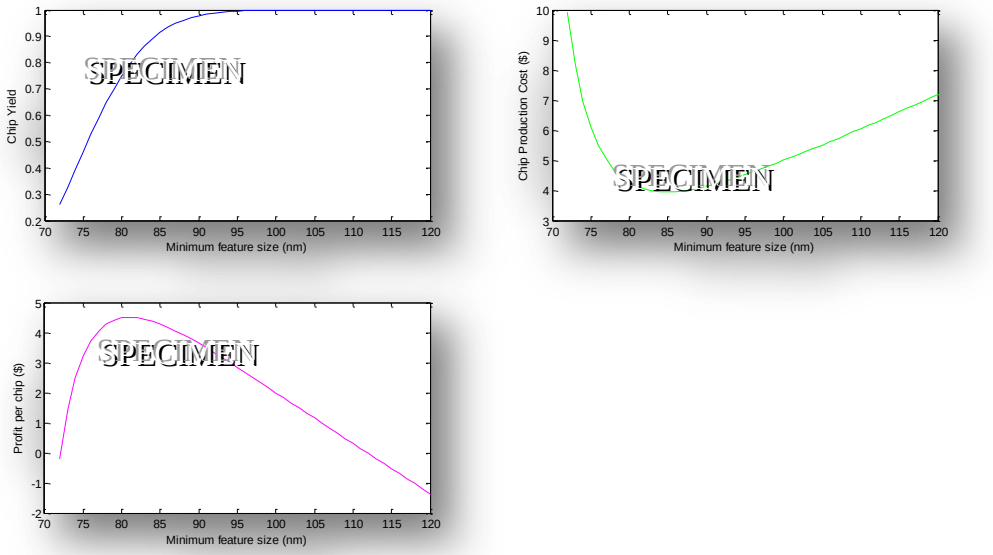
$$Yield = 1 - e^{-(w-w_0)^2/2\sigma^2}$$

$$Production\ Cost = \frac{5x10^{-4}w^2}{Yield} \qquad (1.3)$$

$$Profit = \frac{700}{w} - Cost$$

where w is the feature size in nm, (which must be greater than w_0, the ultimate resolution, in this model), and σ is the sensitivity of Yield to feature size. If w_0 = 65 nm and σ = 9 nm, plot Yield, Production Cost and Profit as a function of feature size, for feature sizes between 72 nm and 120 nm.

You should get something like this.



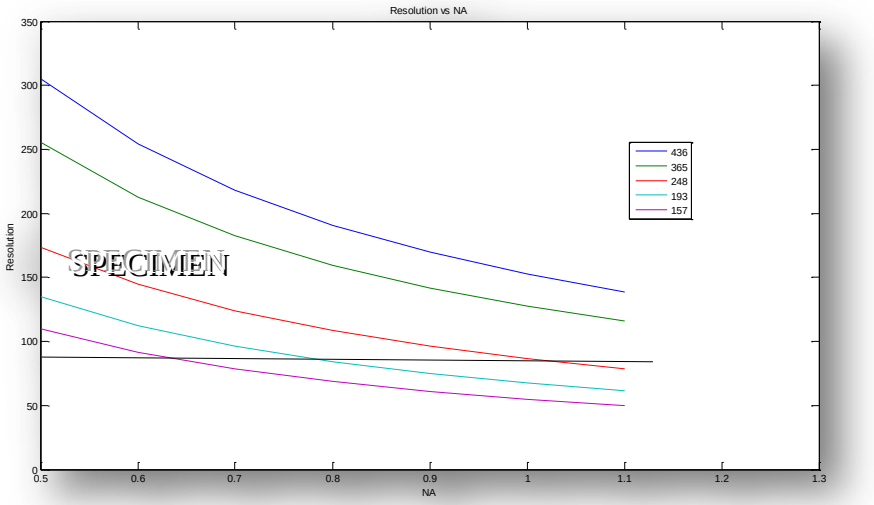
Answer the following questions.

(i) Below what feature size does the Yield drop below 30%?
(ii) Above what feature size does the Yield exceed 95%?
(iii) What is the feature size at the lowest Production Cost?
(iv) What is the Yield at the lowest Production Cost?
(v) What is the maximum Profit per chip, in dollars, at this plant?
(vi) What is the feature size when the plant is most profitable?
(vii) What is the Yield when the plant is most profitable?

3. Resolution in optical lithography scales with wavelength, λ, and numerical aperture, *NA*, according to a modified Rayleigh criterion:

$$R = \frac{k_1 \lambda}{NA}$$

where k_1 can be thought of as a constant for a given lithographic approach and process. Assuming k_1 = 0.35, plot resolution versus numerical aperture over a range of *NA*s from 0.5 to 1.0 for the common lithographic wavelengths of 436 nm, 365 nm, 248 nm, 193 nm, and 157 nm. From this list, what options (*NA* and wavelength) are available for printing 90 nm features? Your plot should look like this.



4. Resist thickness scales as

$$Thickness \propto \frac{\upsilon^{0.4}}{\omega^{0.5}} \qquad (1.4)$$

ω is the spin speed and υ is the resist viscosity. If *k* is the constant of proportionality.

$$Thickness = k\frac{\upsilon^{0.4}}{\omega^{0.5}}$$

The Reynold's number, *Re*, is given by

$$Re = \frac{\omega r^2}{\upsilon_{air}} \qquad (1.5)$$

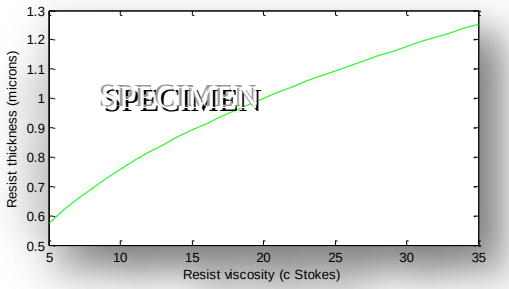
where *r* is the wafer radius and υ_{air} is the viscosity of air.

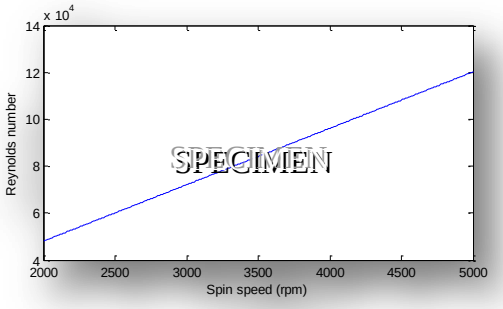
(i) Plot thickness of resist as a function of resist viscosity for viscosities of 5 – 35 cStoke (1 Stoke = 1 cm^2/s), for a spin speed of 2000 rpm. Take care with dimensions, assuming $k = 4.38 \times 10^{-4}$, υ is in m^2/s and ω is in revolutions per second.

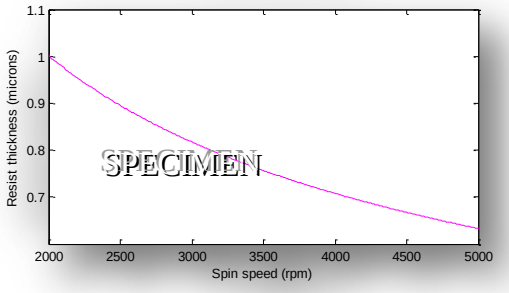
(ii) Plot the Reynolds number as a function of spin speed for a 300 mm wafer and prove that a spin speed of 2000 rpm will not cause turbulent flow, given that flow instabilities would be expected to begin at a Reynolds number of 100,000. Take $\upsilon_{air} = 1.56 \times 10^{-5}$ m^2/s.

(iii) Plot thickness of resist as a function of spin speed, for a resist viscosity of 20 cStokes, for spin speeds between 2000 and 5000 rpm.

You should get something like the plots shown.







2. Aerial Image Formation – The Basics

Aerial Image calculation: - Dense Lines and Spaces

1. This exercise is based on Chapter 2 of the book, in particular the material presented in Section 2.2.5. In this exercise, we will look at the aerial image formed by a dense pattern of lines and spaces as a function of the number of diffraction orders captured by the objective lens. We will assume coherent illumination is used.
The diffraction pattern of an infinite array of lines and spaces is made up of discrete diffraction orders, mathematically represented by delta functions. The image of such a pattern is calculated as the Inverse Fourier Transform of the diffraction orders that make it through the lens, that is, those with spatial frequencies less than NA/λ.

We can show that for a dense array of lines and spaces

$$E(x) = a_0 + 2\sum_{j=1}^{N} a_j \cos(2\pi jx/p) \qquad (2.63)$$

The zeroth order, given by a_0, provides a DC offset to the electric field. Each pair of diffraction orders j = +/- 1, 2, 3, etc., contributes a cosine at harmonics of the pitch p to the image. In order to calculate the coefficients a_j ,

$$a_j = \frac{\sin(j\pi w/p)}{j\pi} \qquad (2.62)$$

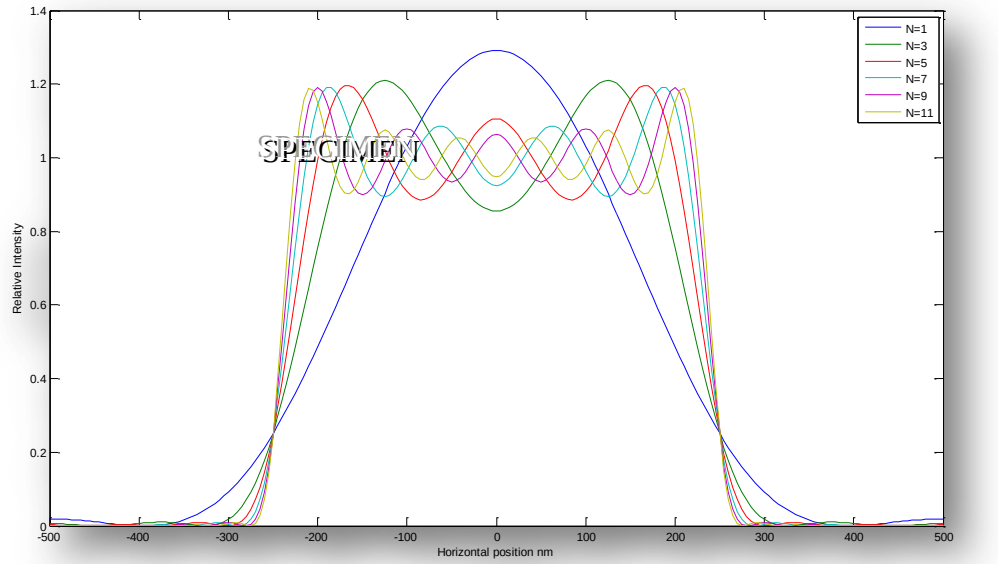
where w is the linewidth and p is the pitch. The Electric Field Intensity is given by

$$I(x) = n|E(x)|^2 \qquad (2.39)$$

Take n = 1 for this exercise, that is, assume that the medium is air.

2. For equal lines and spaces, calculate a_1 to a_{11}. How can a_0 be calculated?

3. Write a MATLAB® program to calculate the Aerial image, that is, the Electric Field Intensity, $I(x)$, across a space, as a function of the number of diffraction orders captured by the objective lens. Assume coherent illumination is used for a pattern of equal lines and spaces of width 500 nm and an associated pitch of 1000 nm. Do the calculation for N = 1, 3, 5, 7, 9, 11.



You should get a plot which looks like the one above.

4. If you repeated the exercise, but this time included even diffracted orders, that is, N = 2, 4, 6, 8 etc., how would the results be affected?

5. Do a hand calculation of the intensity at the centre of the space ($x = 0$) and the centre of a line ($x = p/2$) for the case where N = 1, that is, 3-beam imaging. Does it match the results from your MATLAB® simulation?

6. Note that as the number of orders captured by the lens, that is, the *NA*, increases, the image improves. The ringing in the image is reduced, while the transition from dark to light is sharper. Ideally, you want to collect as many diffracted orders as possible, that is, have a large *NA*. If a wavelength of 248 nm is used to image this dense pattern of lines and spaces, and given that on-axis, coherent illumination is used, calculate which diffraction orders pass through the lens. Take the numerical aperture of the lens to be 0.85. Plot the image using MATLAB®.

7. In the text, Section 2.3.1, it says that oblique illumination results in a shift of the diffraction pattern by an amount, in spatial frequency terms, of $\sin\theta'/\lambda$. If the illumination system in the stepper described earlier is tilted by an angle of 30 degrees, identify which diffraction orders get through to the lens.

Partially coherent illumination, the Kintner method

1. This Exercise is based on Chapter 2 of the book, in particular the material presented in Section 2.3.2.

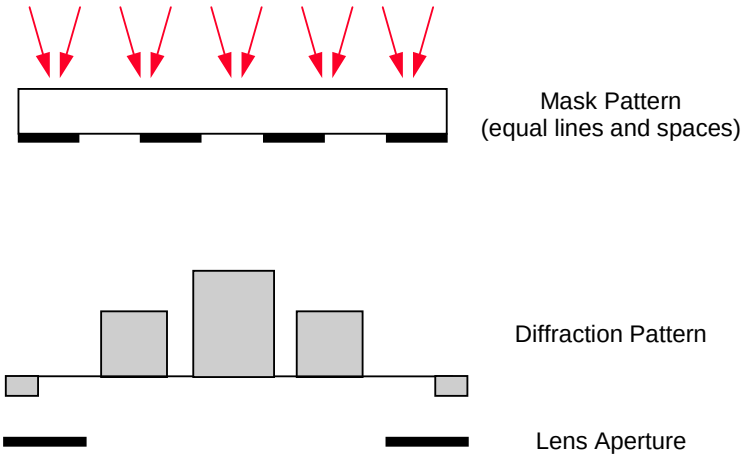


Figure 2.15 *The diffraction pattern is broadened by the use of partially coherent illumination (plane waves over a range of angles striking the mask)*

If the mask illumination comes from a range of angles, rather than just one, we say the light is partially coherent.

For a uniform intensity, circular source and a repeating pattern of lines and spaces, the aerial image is given by

$$I(x) = s_1 I_{1-beam} + s_2 I_{2-beam} + s_3 I_{3-beam} \qquad (2.90)$$

where s_1, s_2, and s_3 are given in Table 2.3 and

$$\chi = \frac{1+\left(\frac{\lambda}{pNA}\right)^2 - \sigma^2}{\frac{2\lambda}{pNA}} \qquad (2.91)$$

$$\gamma = \frac{\sigma^2 tan^{-1}\left(\frac{\sqrt{1-\chi^2}}{\chi-\frac{\lambda}{pNA}}\right) - cos^{-1}(\chi) + \frac{\lambda\sqrt{1-\chi^2}}{pNA}}{\pi\sigma^2} \qquad (2.92)$$

$$\eta = \frac{2\left[cos^{-1}\left(\frac{\lambda}{pNA}\right) - \frac{\lambda}{pNA}\sqrt{1-\left(\frac{\lambda}{pNA}\right)^2}\right]}{\pi\sigma^2} \qquad (2.93)$$

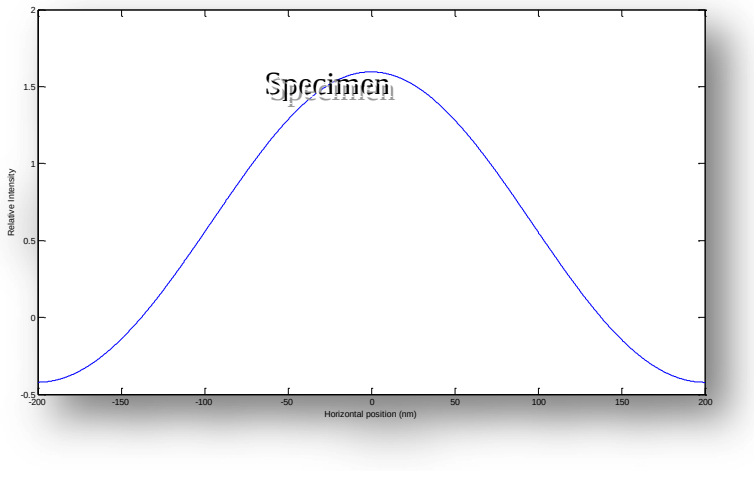
Table 2.3 *Kintner weights for conventional source imaging of small lines and spaces.*

Regime	Description	s_1	s_2	s_3
$\frac{\lambda}{pNA} \le 1-\sigma$	All 3-beam imaging	0	0	1
$1-\sigma \le \frac{\lambda}{pNA} \le \sqrt{1-\sigma^2}$	Combination of two- and three-beam imaging	0	2γ	$1-2\gamma$
$\sqrt{1-\sigma^2} \le \frac{\lambda}{pNA} \le 1$	Combination of one-, two- and three-beam imaging	$\eta + 2\gamma - 1$	$2(1-\gamma-\eta)$	η
$1 \le \frac{\lambda}{pNA} \le 1+\sigma$	Combination of one- and two-beam imaging	$2\gamma - 1$	$2(1-\gamma)$	0
$1+\sigma \le \frac{\lambda}{pNA}$	Only the zero order is captured, all 1-beam	1	0	0

In this Exercise, we will calculate the aerial image formed by a dense pattern of lines and spaces formed using partially coherent illumination.

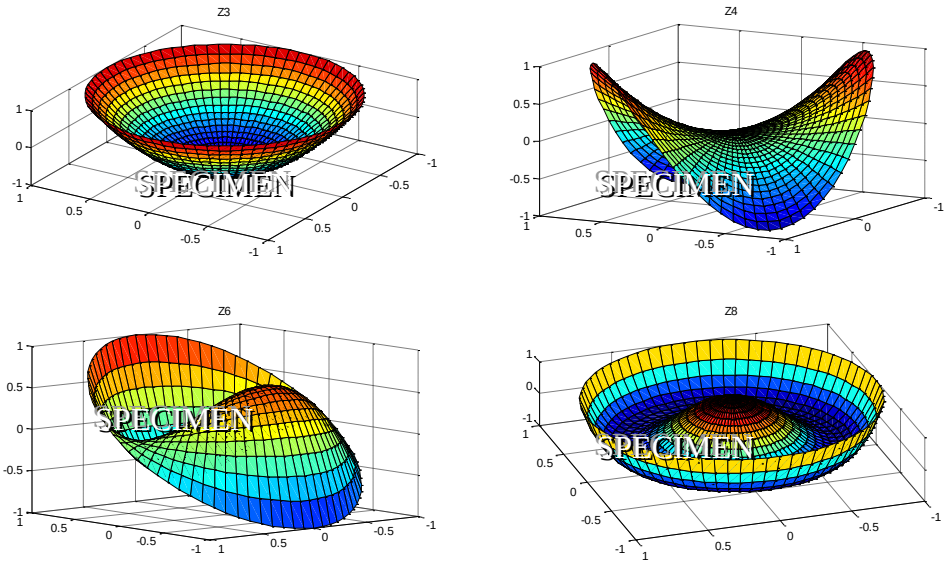
2. Read pp 58-62 in *Fundamental Principles of Optical Lithography*, by Chris Mack.

3. Write a MATLAB® program to calculate the Aerial image, that is, the Electric Field Intensity, *I(x)*, across a space. Assume partially coherent, 248 nm illumination with values of σ = 0.8, *NA* = 0.8 and the linewidth = spacewidth = 200 nm. Note that this involves plotting Eq (2.90). You should get a plot which looks like the one below.



3. Aerial Image Formation – The Details

1. Plot the *Z3*, *Z4*, *Z6* and *Z8* Zernike formulae using MATLAB®. You should get something like this.



2. Equation (3.6) describes 3rd order *x* coma as

$$Z_6\,(3R^2-2)RcosΦ \qquad (3.6)$$

For simplicity, we consider coma in the direction of maximum magnitude is given by Eq (3.6) with $cosΦ$ maximum, that is 1. From Table 3.1, $Z_1\ RcosΦ$ is the x-tilt. Comparing these Equations, we see that coma acts as an x- tilt with coefficient $Z_6(3R^2-2)$. This leads to a placement error which varies with *R*.

We know that tilt generates a placement error of

$$Placement\ error = -\frac{Z_1\lambda}{NA} \qquad (3.5)$$

By comparison, we would expect coma to generate a placement error of

$$Placement\ error = -\frac{Z_6\,(3R^2-2)\lambda}{NA}$$

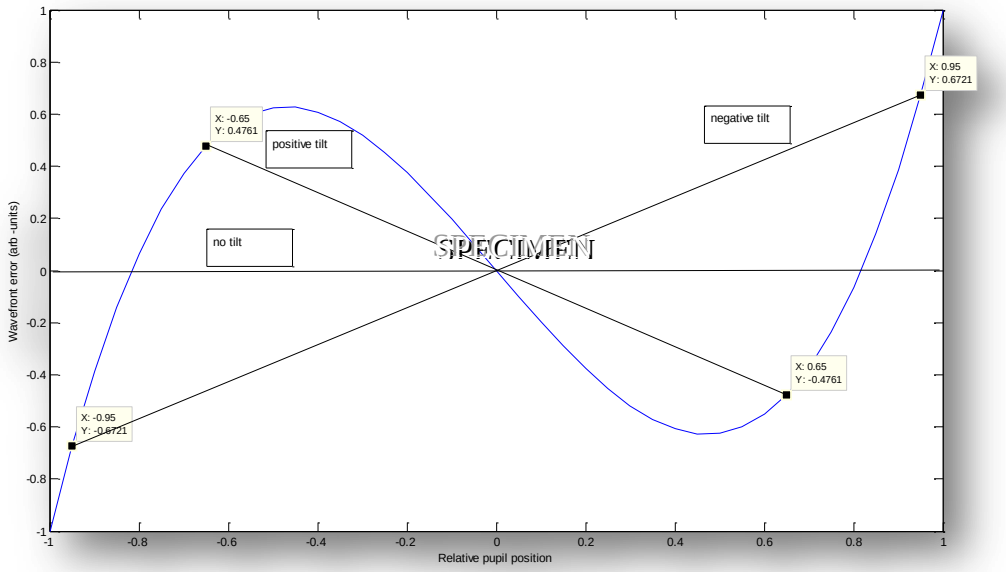
Consider a pattern of lines and spaces of pitch p. Diffraction will produce discrete orders of light passing through the lens. The zeroth order goes straight through the lens centre, while the first orders enter the lens at radial positions $R = \pm\lambda/pNA$. The placement error is given by substituting $R = \lambda/pNA$ into the Equation for placement error as a result of Coma above.

$$Placement\ error = -\frac{Z_6\lambda}{NA}\left(3\left(\frac{\lambda}{pNA}\right)^2 - 2\right) \qquad (3.7)$$

This applies to line/space patterns and $\lambda/NA < p < 3\lambda/NA$ and coherent illumination. It predicts that you will get a negative placement error when the pitch is less than $1.5\lambda/NA$, a positive error for pitches greater than this amount, and zero placement error when the pitch is equal to this amount.

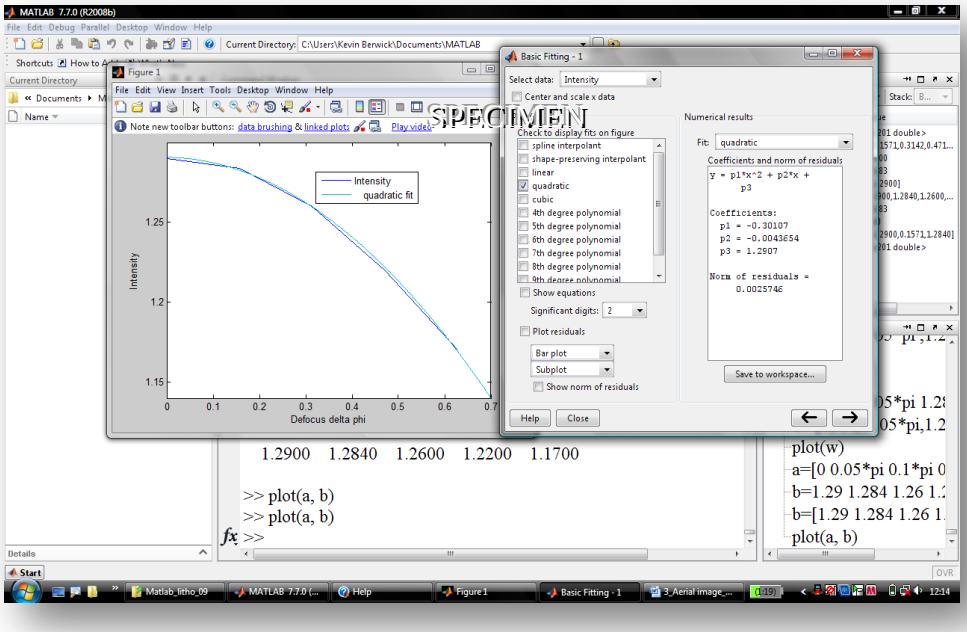
The error is zero when $(3R^2 - 2) = 0, or\ R = 0.816$, other values of R will suffer placement error. Show why this is true, using plots of the wavefront error generated using MATLAB®.

You should get something like this for your plot of wavefront error.



3. Consider the case of dense equal lines and spaces (only the 0 and ±1st orders are used) imaged with coherent TE illumination. Using MATLAB®, plot the aerial image intensity for this situation, given by Eq (3.27). By changing the amount of defocus over a suitable range, show that the peak intensity of the image in the middle of the space falls off approximately quadratically with defocus, for small amounts of defocus. Here is some typical data, together with a fit generated using MATLAB®.

Defocus	**Intensity**
0	1.29
0.05π	1.284
0.1 π	1.26
0.15 π	1.22
0.2 π	1.17

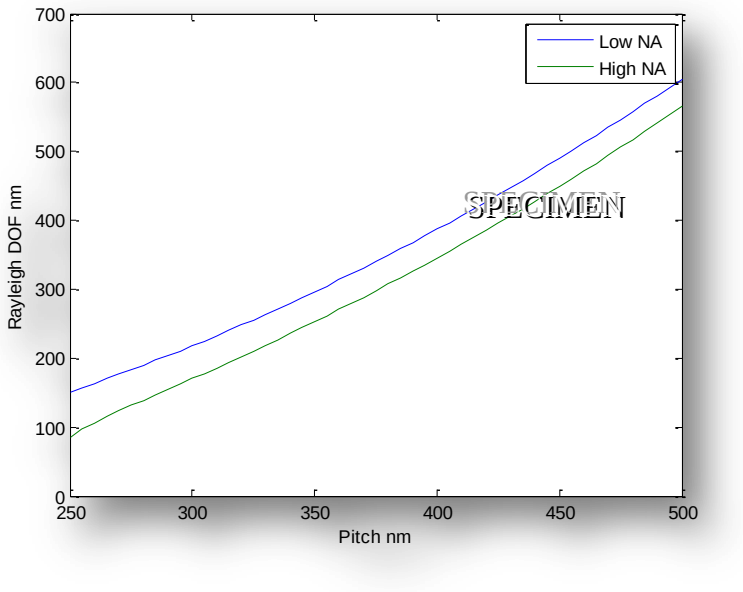


4. Compare the depth of focus predictions of the high-NA version of the Rayleigh equation (3.60) to the paraxial version of equation (3.61) by plotting predicted DOF versus pitch. Use k_2 = 0.6, λ = 248 nm and pitches in the range from 250 to 500 nm. Assume imaging in air.

$$DOF\ (high\ NA) = \frac{k_2}{2}\frac{\lambda}{n(1-\cos\theta)} \qquad (3.60)$$

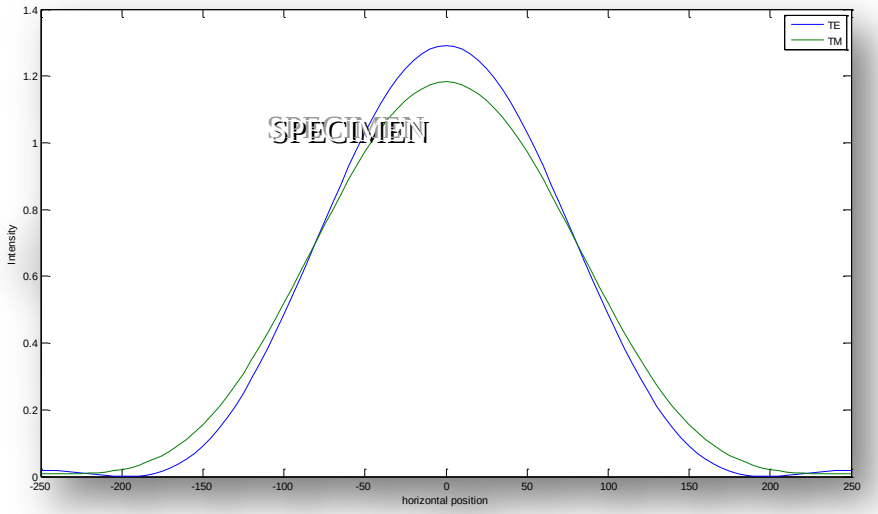
$$DOF\ (low\ NA) \approx k_2\frac{p^2}{\lambda} \qquad (3.61)$$

You should get something like this plot.



5. Read Section 3.6.2. Note that for TE illumination, and for a y - oriented dense pattern, the electric field component for TE polarisation is in the y - direction. However, TM illumination has *x*- and *z*- components present, and will give a degraded image when compared to that produced by TE illumination.

Consider the case of dense equal lines and spaces (only the 0 and ±1st orders are used), with the linewidth and spacewidth equal to 250 nm, imaged with conventional coherent illumination at a wavelength of λ = 193 nm. Explore the influence of polarisation of the illumination source by plotting both the TE and TM aerial images.
You should get something like this.



6. In this exercise, we will look at the effect of defocus on a pattern of lines and spaces formed by a projection lithography system utilizing the 0 and +/-1 diffraction orders only, that is, 3-beam imaging using coherent illumination and TE polarization.

The intensity image in this case is given by

$$I(x) = a_0^2 + 4a_0 a_1 \cos(\Delta\Phi)\cos\left(\frac{2\pi x}{p}\right) + 2a_1^2\left[1 + cos\left(\frac{4\pi x}{p}\right)\right] \quad (3.27)$$

The impact of defocus is to reduce the magnitude of the main cosine term in the image. As the first orders go out of phase with respect to the zeroth order, the interference between the zeroth and first orders decreases. When $\Delta\Phi = 90^0$, there is no interference and the $\cos\left(\frac{2\pi x}{p}\right)$ term disappears. For defocus δ we get

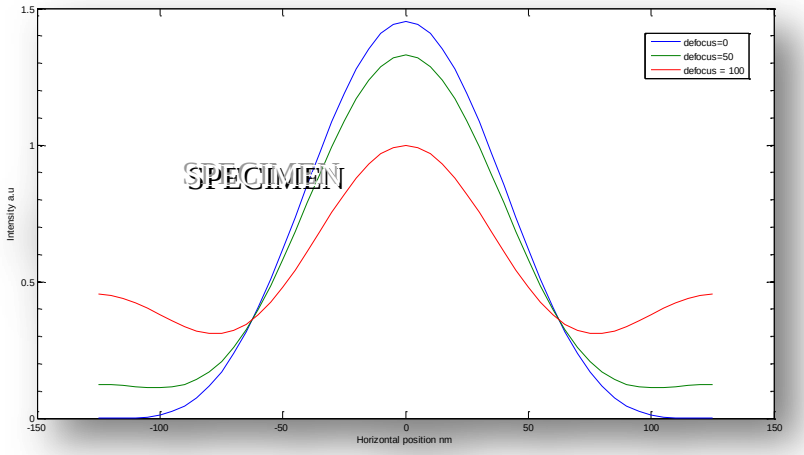
$$\Delta\Phi = \frac{2\pi n\delta\left(1-\sqrt{1-\left(\frac{\lambda}{np}\right)^2}\right)}{\lambda} \quad (3.25)$$

Note that for coherent imaging, $\sigma = 0$.

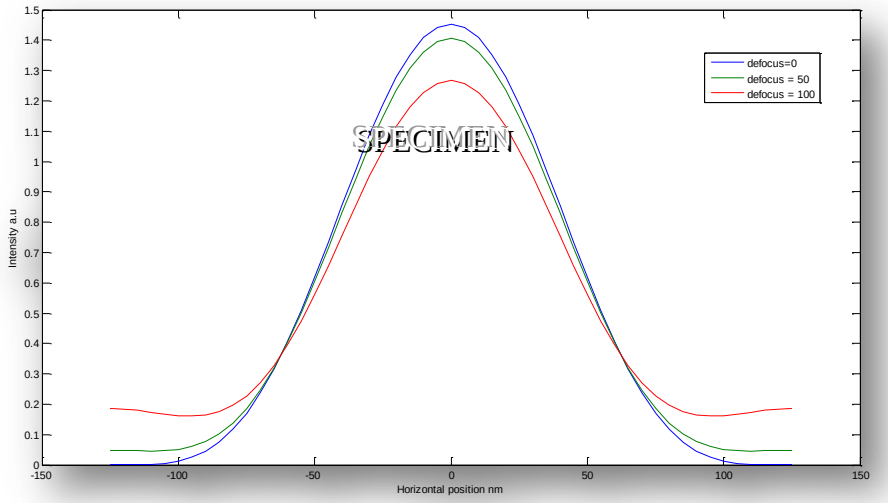
Consider a coherent, three-beam image of a line/space pattern under TE illumination. *NA* = 0.93, λ = 193 nm, linewidth = 100 nm, spacewidth = 150, pitch = 250 nm. Plot the resulting aerial image at best focus and at defocus values of 50 and 100 nm for:

i) imaging in air
ii) immersion imaging in a fluid with a refractive index of 1.44.

You should get something like this for imaging in air.



You should get something like this for immersion imaging.



7. (i) Consider an in-focus, TE image where only the zeroth order and one of the first orders pass through the lens. This is two-beam imaging. For this image, what are the values of β_0, β_1, and β_2 in Equation (3.93)?

$$I(x) = \sum_{j=0}^{N} \beta_j \cos\left(\frac{2\pi j x}{p}\right) \qquad (3.93)$$

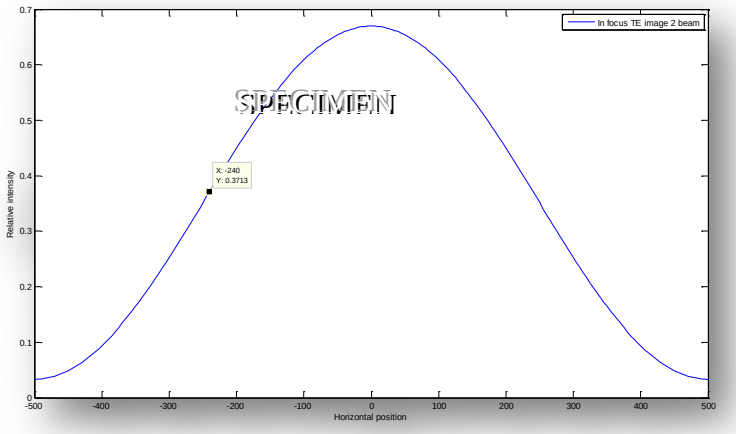
(ii) Using Eq (3.96), show that the value of the *NILS* for the case of equal lines and spaces is 2.85.

(iii) Plot the aerial image for equal lines and spaces of width 500 nm. Measure the *NILS* by measuring the slope of the image. Use the formula

$$NILS = \frac{w}{I}\frac{dI}{dx} \quad \text{with } \frac{dI}{dx} \text{ being the slope of the Aerial image}$$

and compare it with the theoretical result from part (ii).

Your plot should look like this.



8. For the line/space image of equation (3.93):

$$I(x) = \sum_{j=0}^{N} \beta_j \cos\left(\frac{2\pi j x}{p}\right) \qquad (3.93)$$

i) Derive an equation for the image contrast, defined as

$$Image\ contrast = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad (9.1)$$

For simplicity, assume the maximum intensity occurs in the middle of the space, and the minimum intensity occurs in the middle of the line.

ii) For the case where the second harmonic is the highest harmonic in the image (i.e., when N = 2 in Eq (3.93)), compare the expression for the resulting Image contrast to the *NILS* of equation (3.96). Note that this is 3-beam imaging.

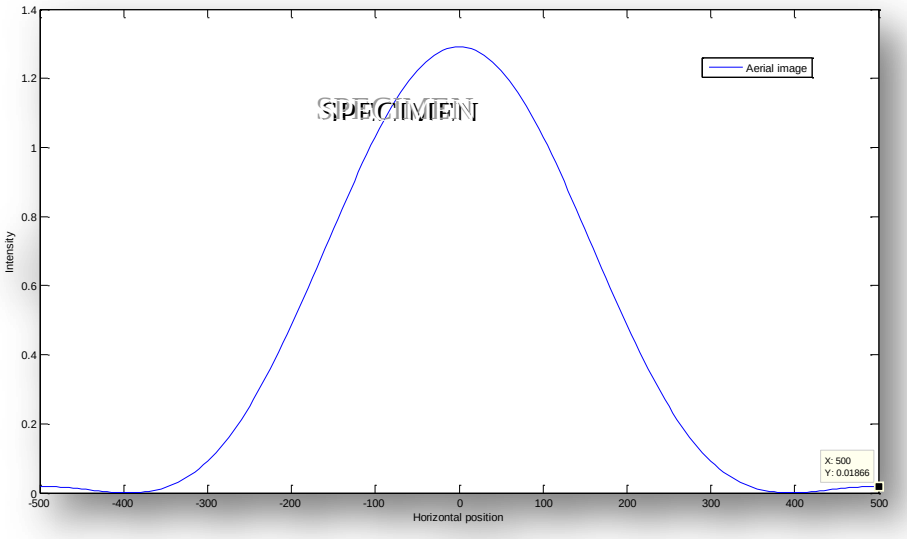
$$NILS = -\pi \frac{\beta 1}{\beta 0 - \beta 2} \qquad (3.96)$$

(iii) Consider an in focus, three-beam, TE image with an aerial image intensity described by the equation

$$I(x) = a_0^2 + 2a_1^2 + 4a_0\, a_1 \cos\left(\frac{2\pi x}{p}\right) + 2a_1^2 cos\left(\frac{4\pi x}{p}\right) \qquad (3.69)$$

(a) From Table 3.2, what are the values of $\beta_0, \beta_1, \beta_2$?

(b) Plot the aerial image, given by equation (3.69), for equal lines and spaces of width 500 nm. You should get something like this.



(c) Measure the *NILS* from the aerial image plot and compare it to the theoretical *NILS* value of 8 predicted for this imaging situation. In addition, compare it to the theoretical *NILS* from equation (3.96) below.

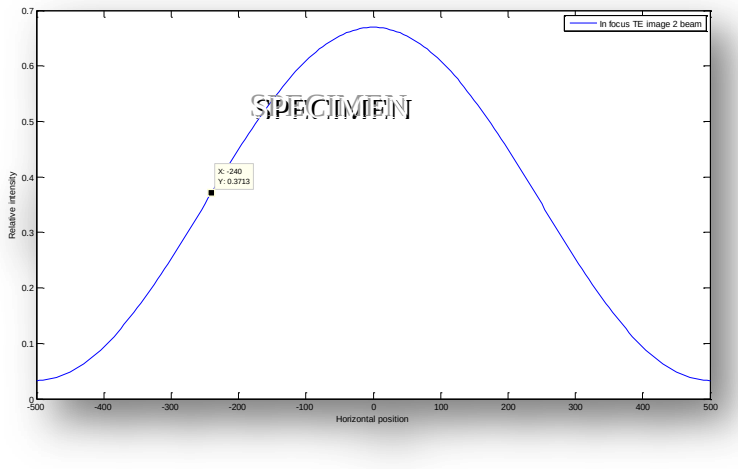
$$NILS = -\pi \frac{\beta 1}{\beta 0 - \beta 2} \qquad (3.96)$$

(d) Measure the image contrast from the aerial image and compare it to the value of $\frac{\beta_1}{\beta_0+\beta_2}$ you would expect.

9. Replot the TE aerial image, however, this time for $N = 1$, that is, assume that only the zeroth and one first order contributes to the image. This is two-beam imaging and requires plotting Eq (3.38).

$$I(x) = a_0^2 + a_1^2 + 2a_0a_1\cos\left(\frac{2\pi x}{p}\right) \qquad (3.38)$$

You should get something like this.



What are $\beta_0, \beta_1, \beta_2$ for this case? Measure the *NILS* from this new image and the Image Contrast. Compare your result to those obtained from equation (3.95) and from the equation for image contrast in terms of β. Compare your results to those obtained in 8 (c) and 8 (d) above. Comment on the impact of the second harmonic on the *NILS* and Image Contrast.

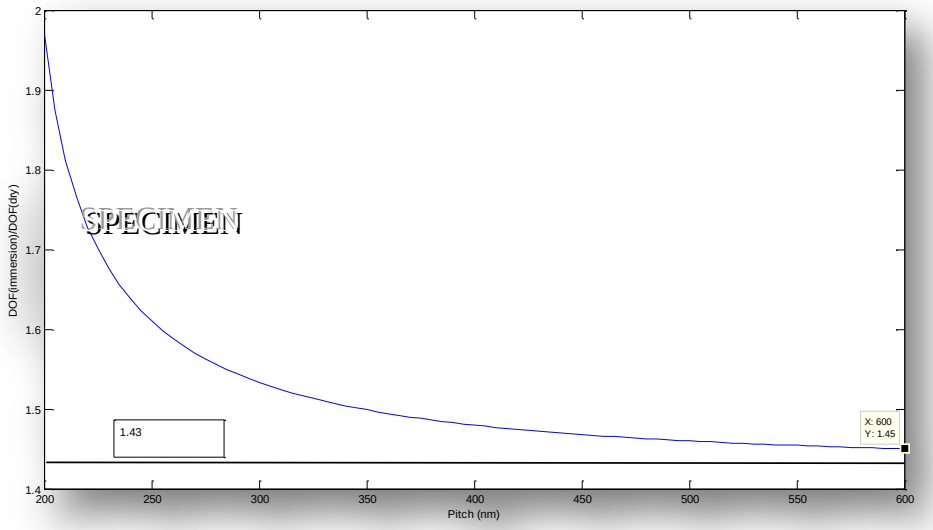
Some typical results are shown below.

	Two beam imaging	**3 beam imaging**
NILS	$NILS=-\pi\frac{\beta 1}{\beta 0}=2.84$	$NILS=-\pi\frac{\beta 1}{\beta 0-\beta 2}$ 8
Contrast	$\frac{\beta 1}{\beta 0}=0.906$	$Image\ contrast=\frac{\beta_1}{\beta_0+\beta_2}$ 0.9715

10. Plot the improvement in Depth of Focus as a result of using immersion lithography, $\frac{DOF_{immersion}}{DOF_{dry}}$. Perform the plot for a binary mask pattern of lines and spaces as a function of pitch, for pitches between 200 nm and 600 nm. The fluid used is ultrapure degassed water with a refractive index of 1.43. The illumination source has a wavelength of 193 nm. Prove that the ratio $\frac{DOF_{immersion}}{DOF_{dry}}$ is at least equal to the refractive index, over this range of pitches. Note that this requires plotting Eq (3.83).

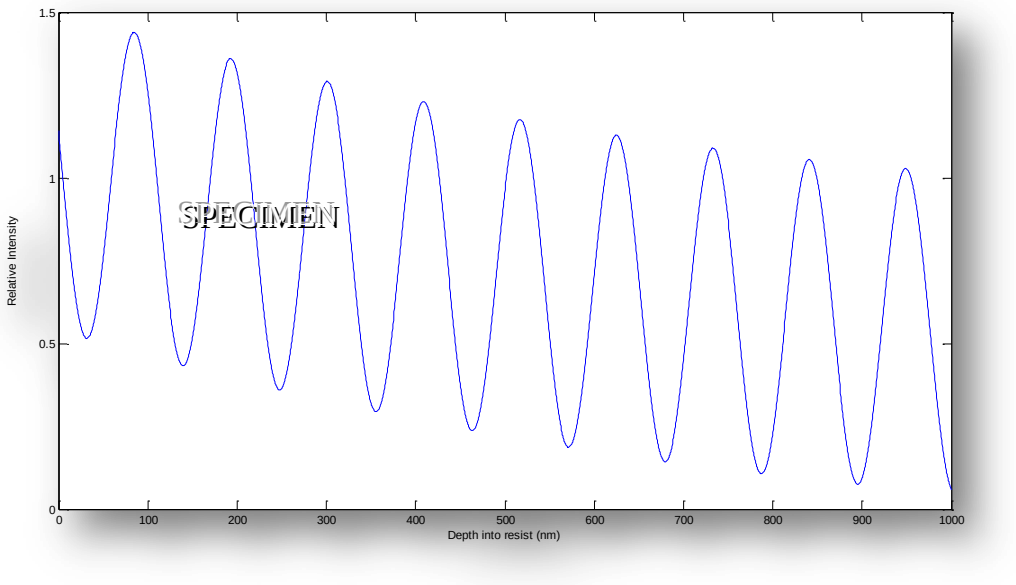
$$\frac{DOF_{immersion}}{DOF_{dry}}=\frac{1-\sqrt{1-\left(\frac{\lambda}{p}\right)^2}}{n-\sqrt{n^2-\left(\frac{\lambda}{p}\right)^2}} \qquad (3.83)$$

You should get something like this.



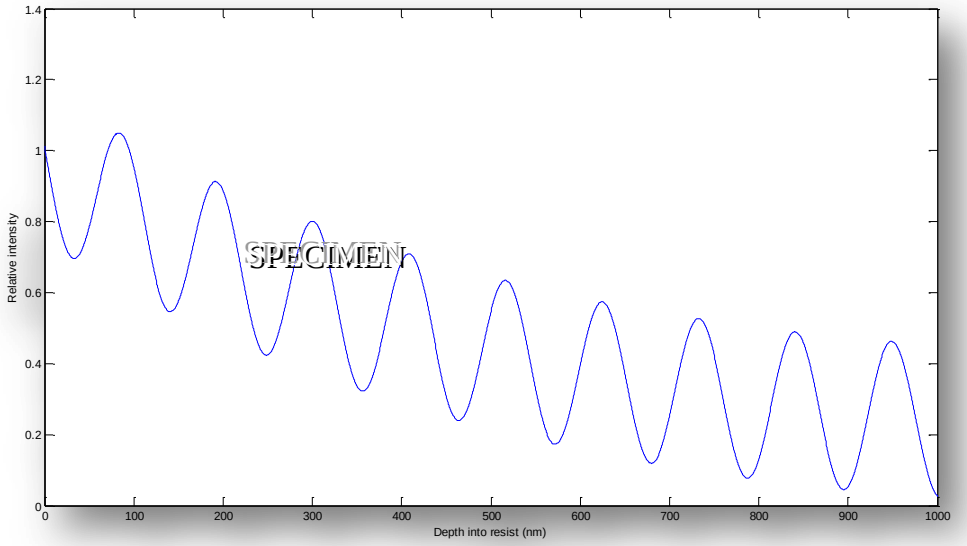
4. Imaging in Resist: Standing Waves and Swing Curves

1. Plot Eq (4.19) from the text, the standing wave intensity as a function of depth, for a 1 micron resist film and i-line illumination at 365 nm. Take the resist refractive index to be *1.69+i0.027* and the silicon refractive index to be *6.5+i2.61*, from Table 4.3. Assume the imaging takes place in air. From the plot, prove that the period of the standing wave = $\frac{\lambda}{2n_2}$, where n_2 is the real part of the resist refractive index. You should get something like this.

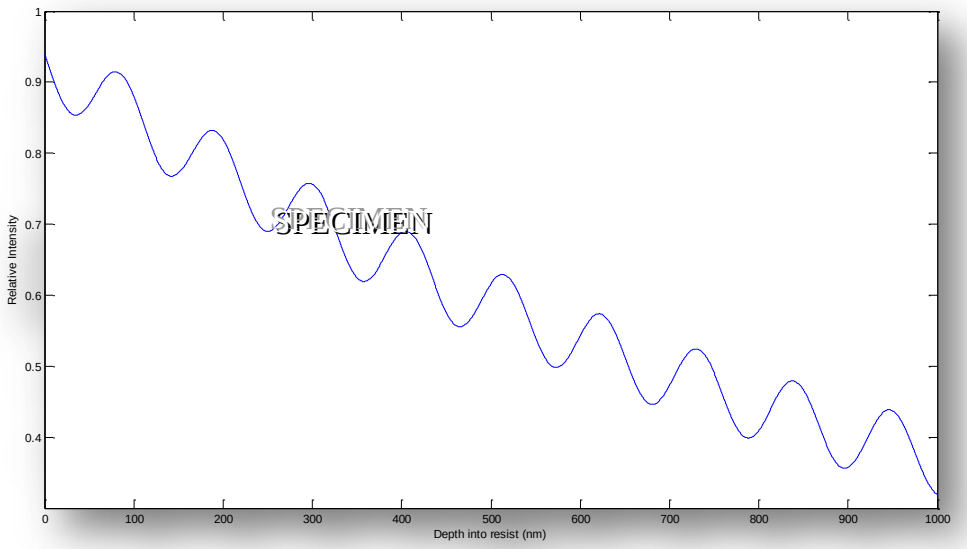


2. Using the program written for Q1, we can explore the use of two techniques which can be used to reduce standing waves in lithography.

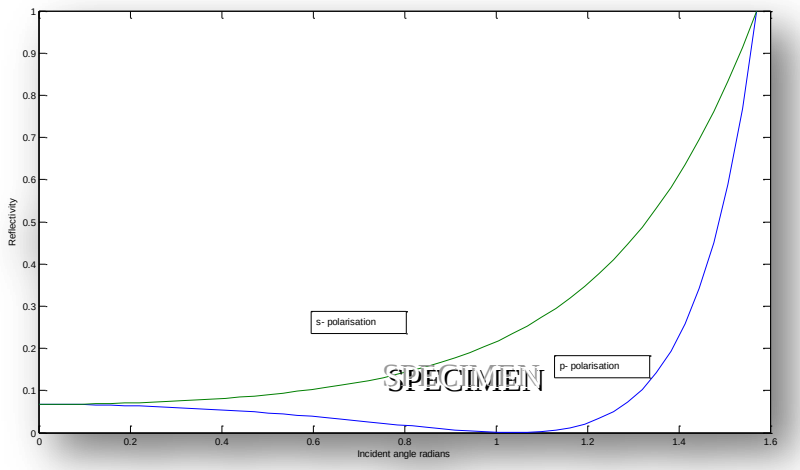
a) Modify the value of the absorption coefficient of the resist by increasing the imaginary part of the refractive index κ to 0.05 for the resist. Physically, this is usually accomplished by adding a dye to the resist to make it more absorbent. Replot the standing wave, giving the diagram shown below. Comment on the result.



b) Simulate the effect of adding a Bottom Anti Reflection Coating (BARC) by reducing the value of ρ_{23} . One straightforward way to simulate this is to simply replace the silicon substrate with a, less reflective, silicon dioxide substrate with a refractive index at 365 nm of *1.474 + i0*. Remember to reset $n_2 = 1.69 + i0.027$. Replot the standing wave giving the diagram shown below. Comment on the result.



3. Plot the reflectivity as a function of angle of incidence for both *s* - polarisation and *p* - polarisation for a beam incident on a resist film with a refractive index of 1.7. Assume the medium is air with a refractive index of 1. Note that this effectively involves plotting Eq (4.29), giving the plot below.



4. For a typical resist on silicon at an illumination wavelength of 248 nm, $\alpha = 0.5\ \mu m^{-1}$, $|\rho_{12}| = 0.275, |\rho_{23}| = 0.73$, $\varphi_{12} = -179.5^{o}$, $\varphi_{23} = -134^{o}$.

Top_first_terms+Top_cosine_arg X Top_cosine

$$R = \frac{|\rho_{12}|^2 + |\rho_{23}|^2 e^{-\alpha 2D} + 2|\rho_{12}\rho_{23}|e^{-\alpha D}\cos(4\pi n_2 D/\lambda - \phi_{12} + \phi_{23})}{1 + |\rho_{12}\rho_{23}|^2 e^{-\alpha 2D} + 2|\rho_{12}\rho_{23}|e^{-\alpha D}\cos(4\pi n_2 D/\lambda + \phi_{12} + \phi_{23})} \quad (4.46)$$

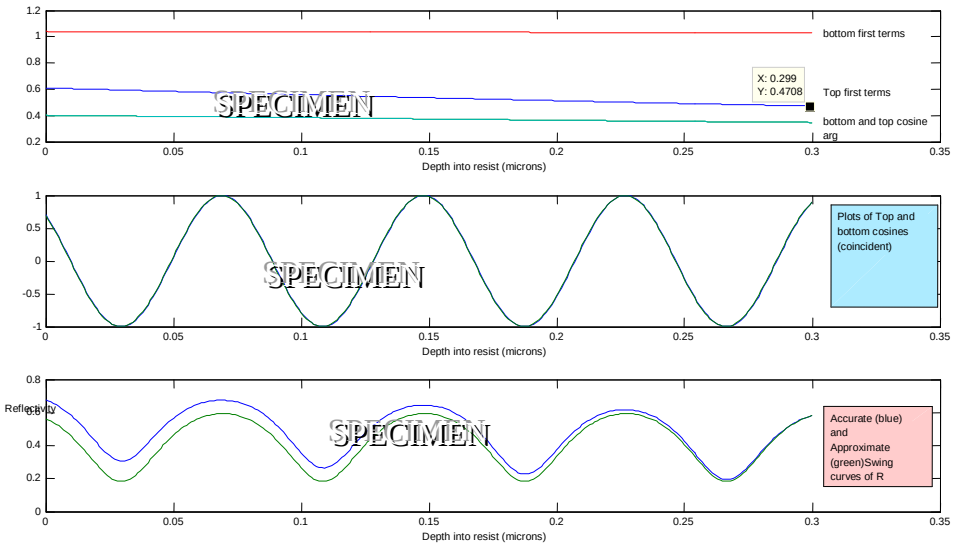
Bottom_first_terms +Bottom_cosine_arg X Bottom_cosine

Use the notation above to refer to the terms in Eq (4.46). Using MATLAB®

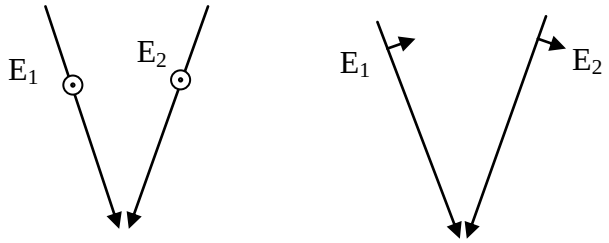
i) Plot the Reflectivity, *R*, as a function of depth, *D*, into the resist using Eq (4.46)
ii) Plot Top_first_terms, Top_cosine_arg and Top_cosine as a function of *D*.
iii) Show that Top_first_terms and Top_cosine_arg vary only slightly with depth, when compared to Top_cosine.
iv) Plot Bottom_first_terms, Bottom_cosine_arg and Bottom_cosine as a function of *D*.
v) Show that Bottom _first_terms and Bottom _cosine_arg vary only slightly with depth, when compared to Bottom _cosine.

vi) Replace Top_first_terms, Top_cosine_arg, Bottom_first_terms and Bottom_cosine_arg in Eq (4.46) with their values from the graphs at a nominal 300 nm. You should get values of 0.47, 0.345, 1.03 and 0.345 respectively. This is now Eq (4.47). Plot the approximate *R*, Eq (4.47), on the same scale as the *R* from Eq (4.46) and comment on the result.
vii) Show that the maximum Reflectivity at a depth of 300 nm is 0.59 and the minimum Reflectivity is 0.18. Calculate the Swing Ratio at this point.

Some sample results are given below for guidance.

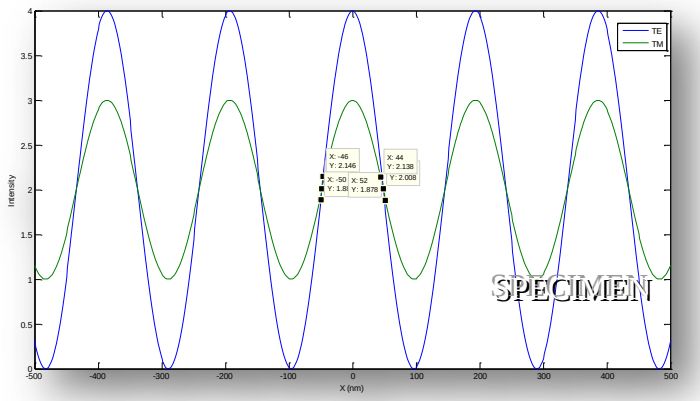


5. Suppose two, unit amplitude, plane waves are travelling with an angle θ to the vertical, as shown below. In this exercise you will plot the TE and TM images that occur as a result of interference, that is, plot Eq (4.109) in the book.



TE or s-polarisation TM or p-polarisation

i) Plot the image for an angle of 30° and for a wavelength of 193 nm. Use *x* values between -500 nm and +500 nm. You should get the diagram below.



ii) What is the period of the resultant waveform?

iii) From the plot, measure the *NILS* and Contrast for the TE image. Show that, for the TE image, the *NILS* is equal to π and the Contrast is equal to 1.

iv) From the plot, measure the *NILS* and Contrast for the TM image. Prove that, for the TM image, the NILS should be equal to $\pi \cos(2\theta)$ and the Contrast should be equal to $\cos(2\theta)$.

v) Increase the angle between the plane waves to 40°. Discuss what happens to the *NILS* and Contrast for both the TE and TM images as a result of increasing this angle.

6. Refer again to the two plane wave interference of Q5. Assuming a resist with a refractive index of 1.7 is used, plot, using MATLAB®, the Contrast as a function of the Angle of the incoming wave for

i) Aerial image with TM illumination.

ii) Resist image with TM illumination.

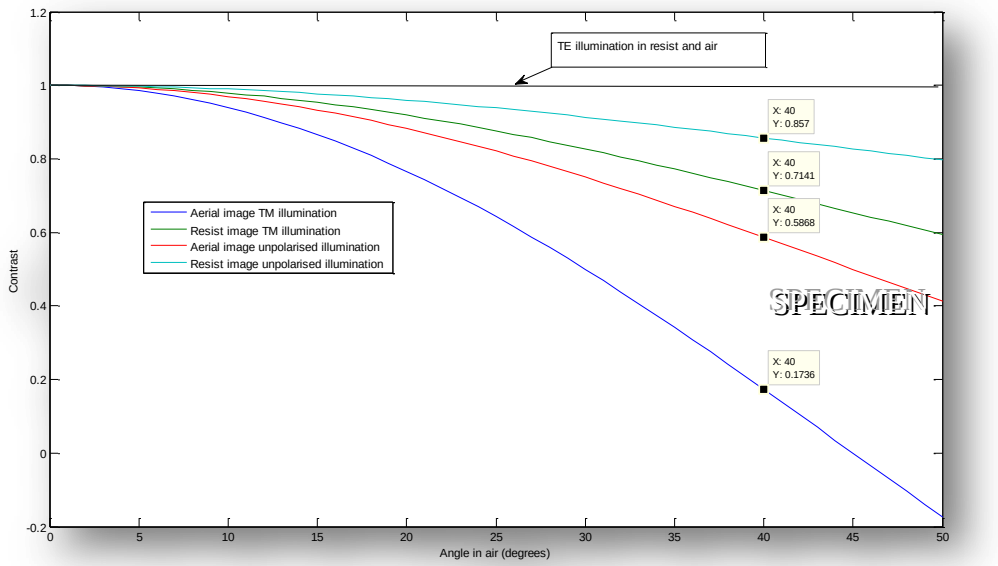
iii) Aerial image with TE illumination.

iv) Resist image with TE illumination.

v) Aerial image with unpolarised light.

vi) Resist image with unpolarised light.

The plot you generate should look like this.



If the angle in air is 40°, calculate the *NA* for two beam imaging at the resolution limit. At this resolution limit,

vii) What is the Contrast for the aerial image with TM illumination?

viii) What is the Contrast for the resist image with TM illumination?

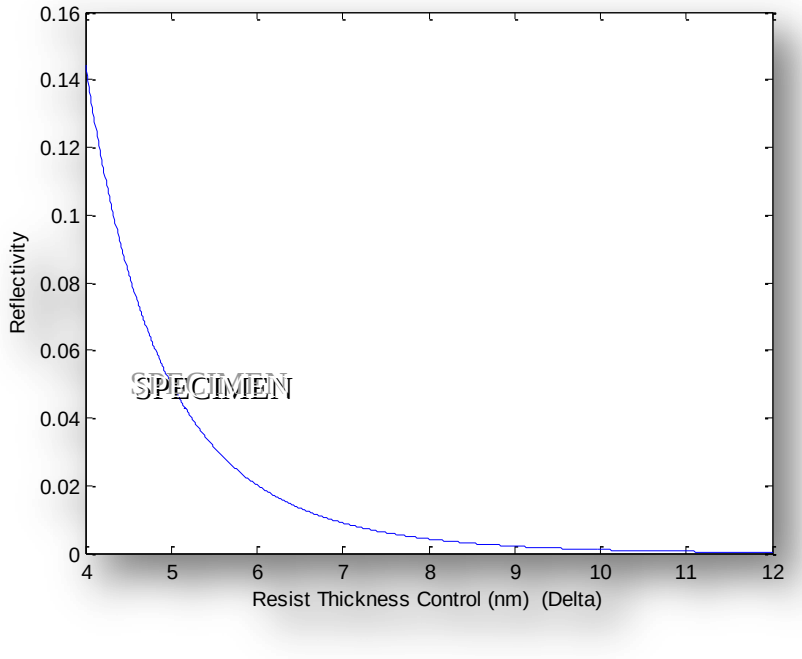
ix) What is the Contrast for the aerial image with TE illumination?

x) What is the Contrast for the resist image with TE illumination?

xi) What is the Contrast for the aerial image with unpolarised illumination?

xii) What is the Contrast for the resist image with unpolarised illumination?

7. As an example of the use of equation (4.100), consider a 193 nm resist with n_2 = 1.7, α = 1.2 μ m^{-1}, D = 200 nm, and $|\rho_{12}|$ = 0.26. Suppose that 2 % dose error due to resist thickness variations is considered acceptable. Generate a plot of maximum allowed substrate reflectivity, $|\rho_{23}|^2$, as a function of the resist thickness variation Δ. You should get something like this.



What is the maximum allowed substrate Reflectivity if the resist thickness increases by 5 nm?

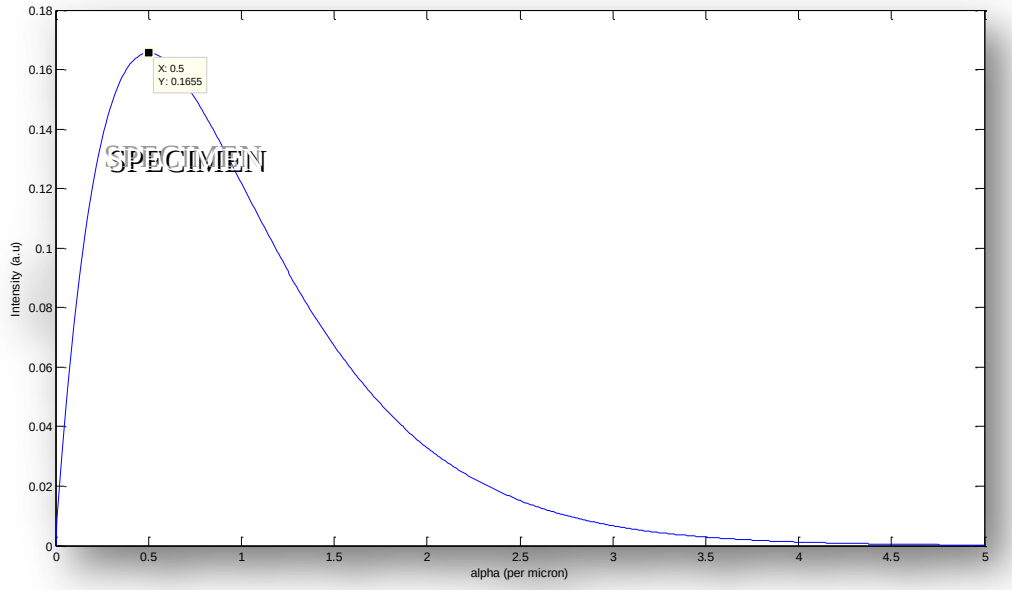
5. Conventional Resists: Exposure and Bake Chemistry

1. For a resist film of a given thickness, there is one value of the absorption coefficient that maximizes the amount of light absorbed by the resist at the bottom of the film. This can be seen by looking at the extremes. When α is zero, all of the incident light makes it to the bottom, but none is absorbed. When α is infinite, no light makes it to the bottom, so none is absorbed there. Thus, in between these two extremes there must be a maximum.
In a homogeneous medium, where α is not a function of *z*, *I(z)* , the Intensity of light at a depth *z* into the resist is given by

$$I(z) = I_0\, e^{-\alpha z} \qquad (5.2)$$

I_0 is the Intensity at $z=0$, that is, at the surface.

Calculate the intensity of light at the bottom of the resist by differentiating Eq (5.2) wrt *z* and noting that at the bottom of the resist, $z=D$. By setting this expression equal to 0 and solving for α, show that the maximum Intensity reaches the bottom of this layer when $\alpha = 1/D$.
Plot the amount of light absorbed at the bottom of a 2 micron resist layer for values of α between 0 μm^{-1} and 5 μm^{-1}. Take I_0, the normalised Intensity incident on the resist, to be 0.9. Show that the maximum Intensity reaches the bottom of this layer when α = 1/D. A typical plot is shown below.



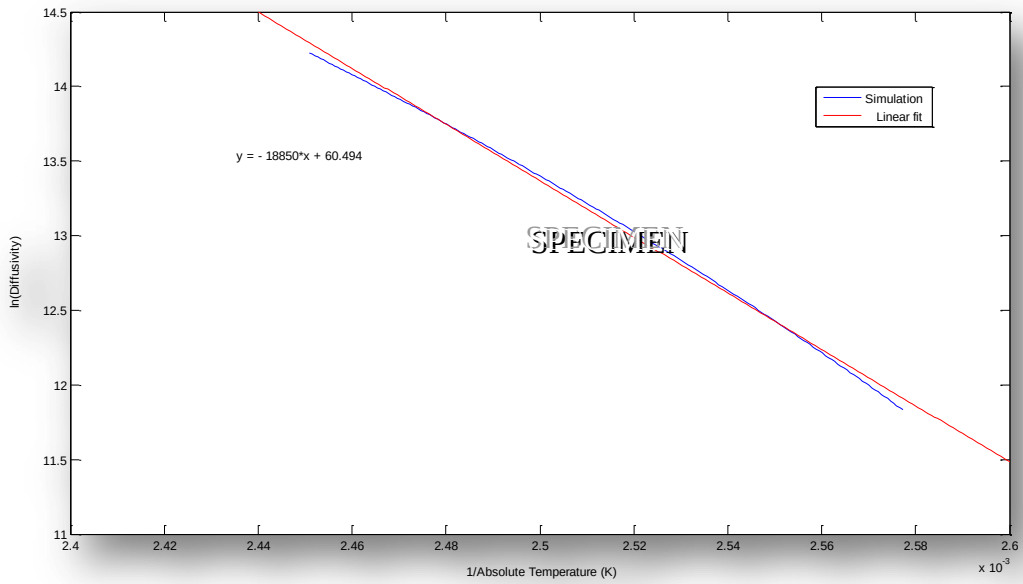
2. The modified Fujita−Doolittle equation (5.61) is shown below.

$$D = D_0 e^{-B\left(\frac{1}{v_f} - \frac{1}{v_g}\right)}$$

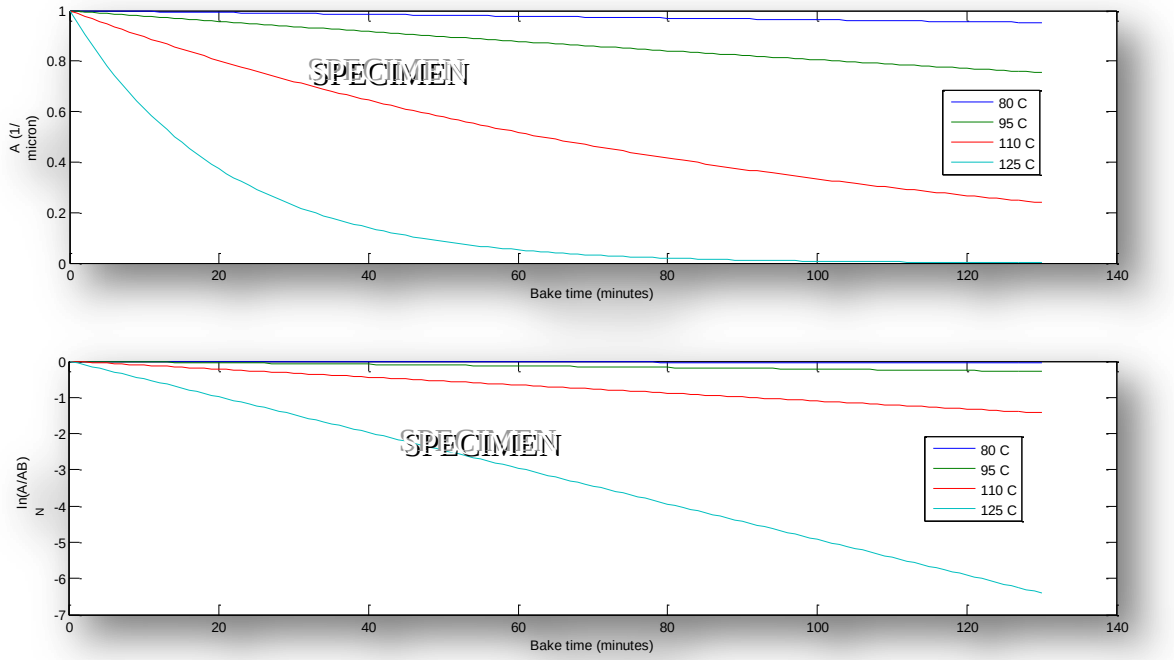
where

$$v_f = (1 - \phi_s)v_g + \alpha_2(T - T_g) + \phi_s\beta \qquad (5.61)$$

It predicts a temperature dependence for solvent diffusivity which is not the Arrhenius equation. For a typical resist, from Table 5.1, B = 0.737, $v_g = 0.0289$, $\alpha_2 = 8.7\ X\ 10^{-4}\ K^{-1}$, $T_g = 110.5\ ^oC$ and $D_0 = 0.0967\ nm^2 s^{-1}$. Take β = 0.351. Assuming 10% solvent volume fraction, create an Arrhenius plot (*ln[diffusivity] vs. 1/T*) for solvent diffusivity from 115 ºC to 135 ºC. How close is this to true Arrhenius behaviour? Find a best estimate for A_r, the Arrhenius coefficient and E_a, the Activation energy, based on this plot. A sample plot is shown below.



3. Using the MATLAB® tools, plot the variation of the resist absorption parameter *A* with post-apply bake time.
Plot *A* versus bake time and $log_e(A/A_{NB})$ versus bake time, for bake times up to 130 minutes. Normalise A_{NB} to 1. Take the Activation energy for the resist to be E_a = 30.3 kcal/mol and the Arrhenius coefficient to be 2.1409 x 10^{15} min^{-1}. *R*, the universal gas constant, is 1.98717 cal/mol-Kelvin.
Assume that decomposition begins immediately the heat is applied, that is, there is no time lag as the wafer ramps in temperature. The plots should look like these.



4. Table 5.1 in the text gives typical diffusion parameters in photoresist. It is possible to use the data in the Table to predict how the solvent diffusivity will vary with temperature. To do this, use the modified Fujita-Doolittle equation to plot *ln (diffusivity)* as a function of *1/Temperature*.
You will need to perform the plot for 2 distinct regimes, *viz*, above the glass transition temperature and below the glass transition temperature. The relevant equations are

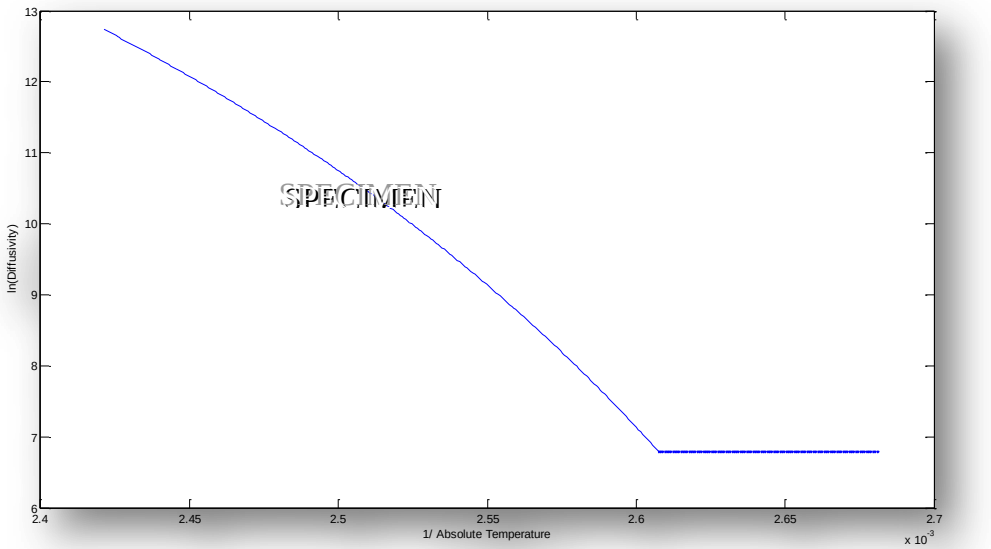
$$D = D_0 e^{-B\left(\frac{1}{v_f}-\frac{1}{v_g}\right)}$$

$$v_f = (1-\Phi_s)v_g + \alpha_2\left(T - T_g\right) + \Phi_s\beta \qquad (5.61)$$

Perform the plot for a value of solvent mass fraction, φ_s, of 0.05. Note that below the glass transition temperature, the value of v_f becomes

$$v_f = (1-\Phi_s)v_g + \Phi_s\beta$$

a consequence of Equation (5.56) . The plot should look like this.



5. A wafer bake, for a 300 mm wafer of thickness 0.775 mm, consists of a 60 second bake on a hotplate at a temperature of 100 ºC, followed by a 10 s transfer to a chillplate at 5 ºC. The wafer remains on the chillplate for 30 s. The ambient temperature is 20 ºC. Assume the initial wafer temperature is the same as that of the ambient. Take the thermal conductivity of air to be k_{air} = 0.031 W/m-K @ 100 ^{o}C. The atomic weight of silicon is 28.09 g/mol. The convection heat transfer coefficient, *h*, is 7.5 W/m^2-K. ρ, the density of silicon is 2.33 g/cm^3. Assume C_p, the silicon molar heat capacitance, is 21 J/K-mol.

Use these equations for this exercise:

$$T^* = \frac{\frac{k_{air}}{\delta}T_{plate} + hT_{ambient}}{\frac{k_{air}}{\delta} + h} \qquad (5.76)$$

$$\tau = \frac{\rho C_p L}{\frac{k_{air}}{\delta} + h} \qquad (5.77)$$

$$T = T^* - (T^* - T_{initial})e^{-t/\tau} \qquad (5.75)$$

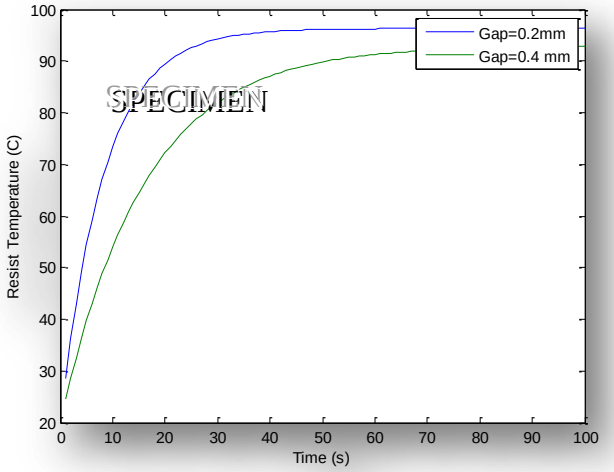
(i) Plot the resist temperature as a function of time for a proximity gap, δ, of 0.2 mm.

(ii) What is the equilibrium wafer temperature (*T**) and the rise time of the bake τ for a proximity gap of 0.2 mm?

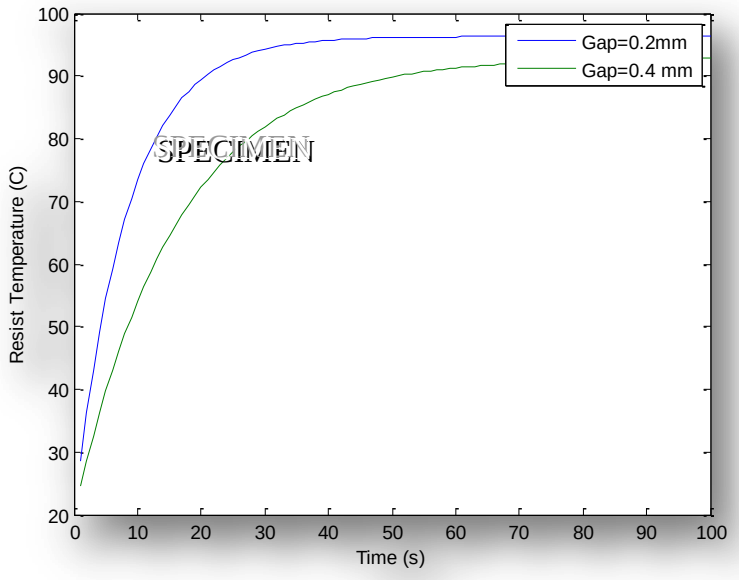
(iii) Plot the resist temperature as a function of time for a proximity gap of 0.4 mm.

(iv) What is the equilibrium wafer temperature (T^*) and the rise time of the bake τ for a proximity gap of 0.4 mm?

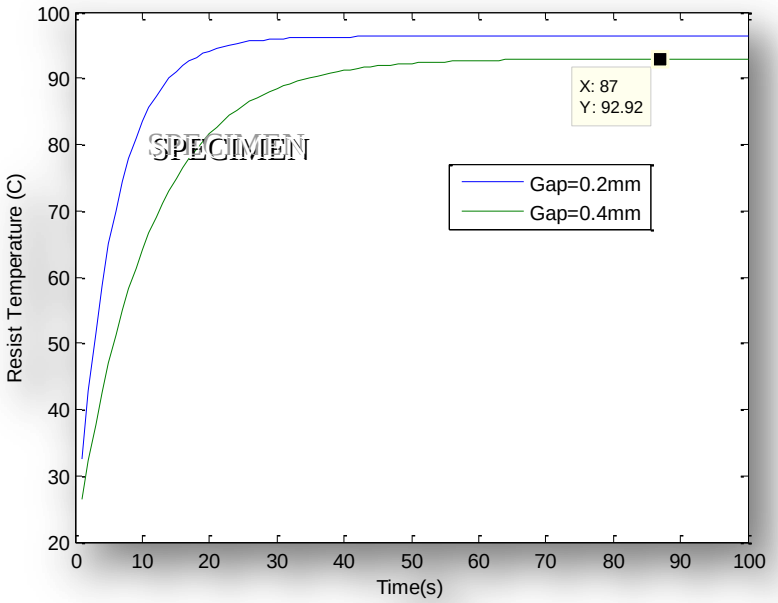
Typical results are shown here.



(v) Redo the plot, returning the proximity gap to 0.2 mm and reducing the wafer diameter from 300 mm to a 100 mm wafer of thickness 0.675 mm. Comment on the impact of this change on the equilibrium wafer temperature (T^*) and the time constant for heating the wafer, τ.
Typical results are shown below.



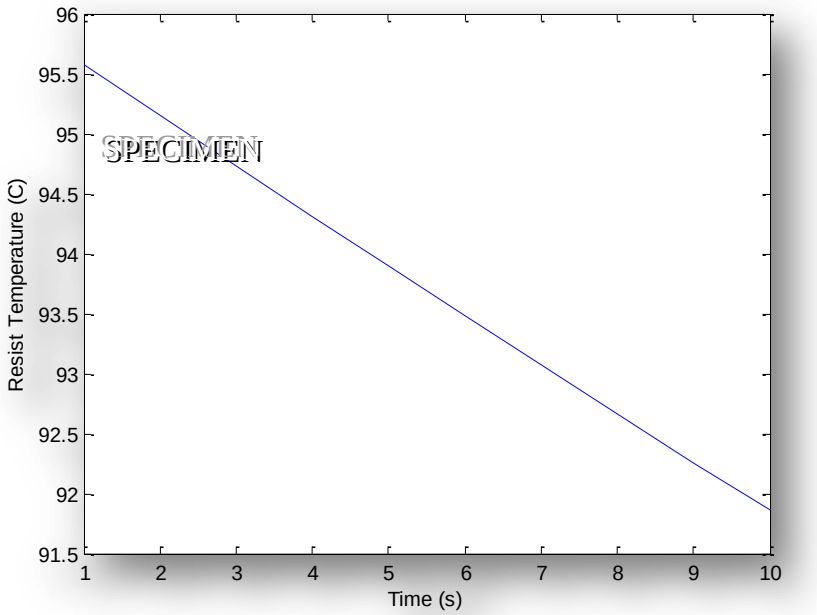
300mm wafer



100mm wafer

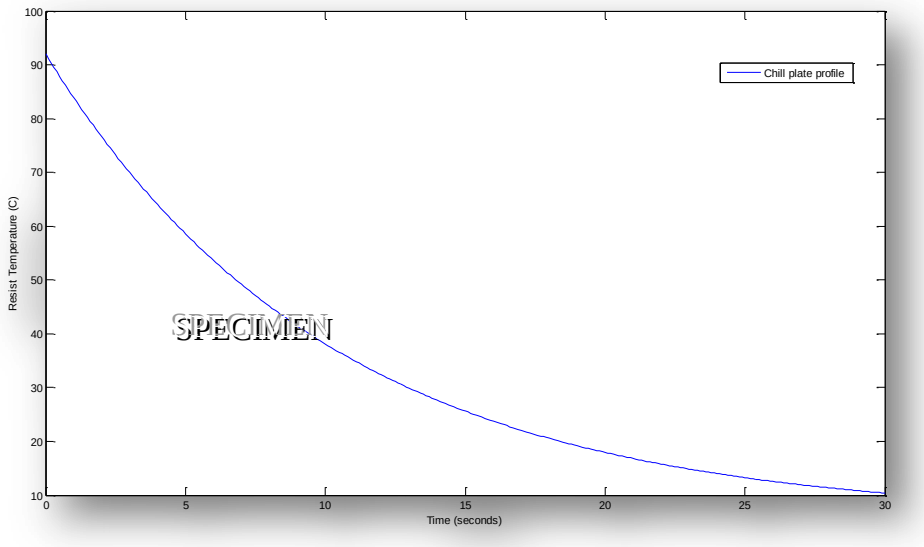
Wafer diameter	**Equilibrium wafer temperature *T** (^{o}C)**	**Time constant for heating (s)**
300mm (0.2mm gap)	96.3	8.3
100mm (0.2mm gap)	96.3	7.23

vi) Plot the temperature profile of the resist on the 300 mm wafer during a 10 s transfer to a chill plate at 5 ^{o}C. Take the initial temperature to be 96 ^{o}C. Typical results are shown



(vii) Plot the temperature profile of the resist on the 300 mm wafer during a 30 s period on a chill plate at 5 ^{o}C. Assume a proximity gap of 0.2 mm during the chill. Take the initial wafer temperature to be 92 ^{o}C.

Typical results are shown below.



6. Tight control of wafer warpage is essential in lithography. This is a result of the limited Depth of Focus in modern steppers. Warpage can also affect other aspects of the lithography flow, as illustrated in this example.

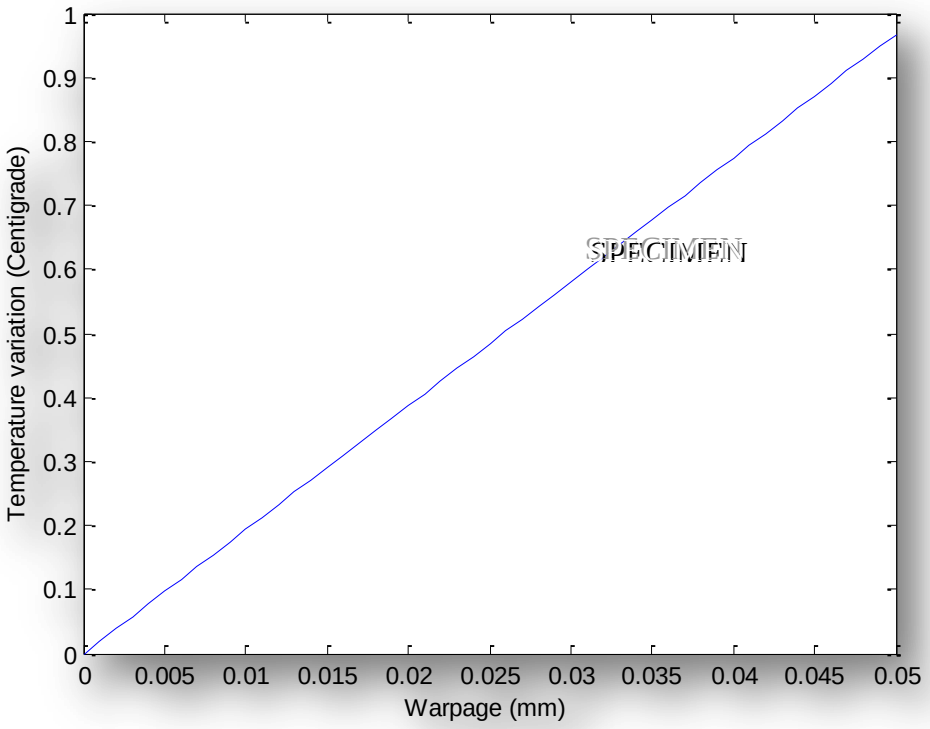
Consider a 300 mm silicon wafer baked at a hotplate temperature of 100 ºC. The proximity gap is 0.2 mm, while the ambient temperature is 20 ºC.

$$T^* = T_{plate} - \frac{\delta h}{k_{air}}\left(T_{plate} - T_{ambient}\right) \qquad (5.79)$$

$$T^* = 100 - \frac{\delta(7.5)}{0.031}(100 - 20) = 100 - (1.935x10^4)\delta \quad \text{Centigrade per metre.}$$

$$= 100 - 19.35\,\delta \;\; Centigrade\ per\ mm$$

The circled term, 19.35δ is the change in equilibrium wafer temperature as a result of varying the proximity gap. Plot the variation in equilibrium wafer temperature as a function of wafer warpage. You should get something like this.



6. Chemically Amplified Resists: Exposure and Bake Chemistry

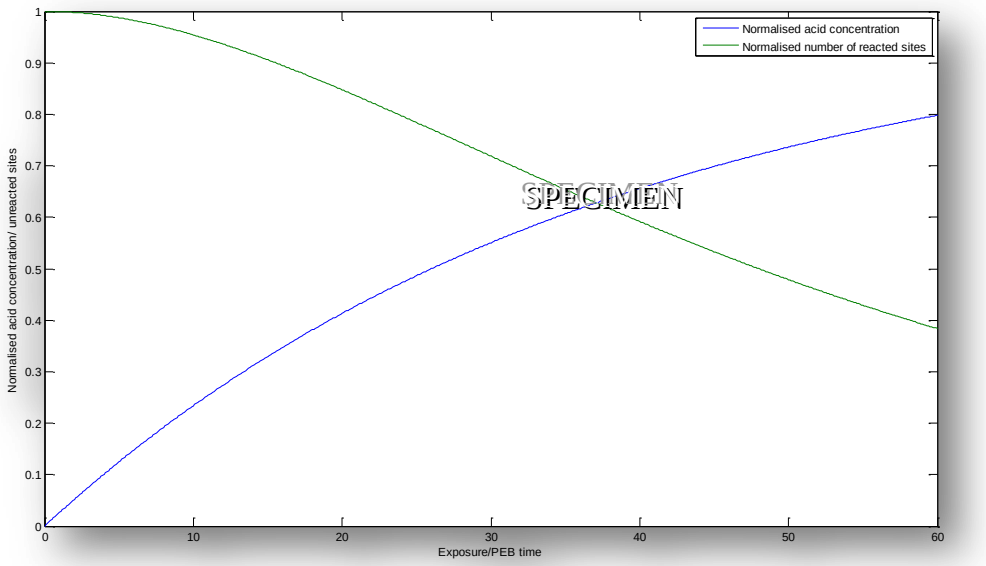
1. A Chemically Amplified Resist receives an exposure dose of 40 mJ/cm^2. The Arrhenius coefficient for the amplification reaction is 3.69 x10^{12} per second, while the Activation Energy is 25 kcal/ mol. Take R = 1.98717 cal/mole-K. The exposure rate constant C for this reaction is 0.04 cm^2 /mJ. Given that the PEB is performed at 110 ^{o}C for 1 minute, calculate the amplification rate constant, K_{amp}, for the reaction.

Plot:

(i) the normalised acid concentration as a function of exposure time.

(ii) the normalised number of unreacted sites as a function of PEB time for this resist.

Assume that the 40 mJ/cm^2 resist exposure is for 60 s and the maximum PEB duration is 60 s, allowing both quantities to be plotted on the same x- axis. You should get a graph which looks like this.



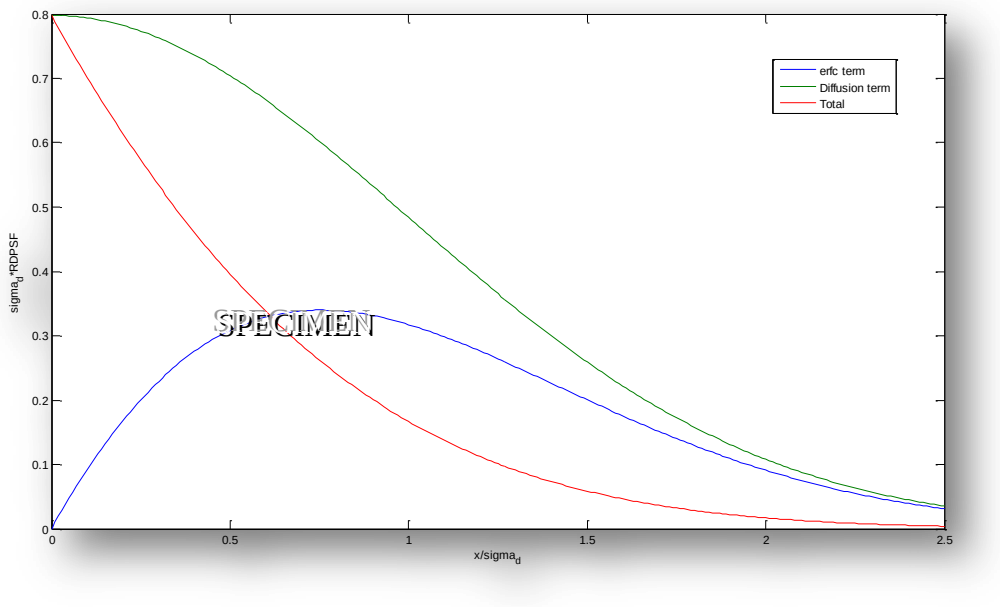
2. The normalised acid concentration, assuming that no acid diffusion occurs, is given by

$$h = 1 - e^{-CIt} \qquad (6.9)$$

This must be modified if we take into account a constant acid diffusivity. Assuming a step function temperature profile for the bake, and ignoring any acid loss, it is possible to solve the reaction rate equation giving,

$$h_{eff} = h(x,0) \otimes RDPSF \qquad (6.17)$$

The acid profile, at time t = 0, is convolved with the Reaction-Diffusion Point Spread Function, RDPSF. Plot the 1D Reaction-Diffusion Point Spread Function for values of x/σ_D between 0 and 2.5. You should get the graph below.



3. Consider a 193 nm exposure of a resist with a Dose-to-Clear of 10 mJ/cm^2. At the resist edge, the mean exposure energy ($= \langle I \rangle T$) will be of the order of the Dose-to-Clear.

(i) Calculate the energy of one photon of light of this wavelength.

For an area of 1 nm x 1 nm.
(ii) Calculate the mean number of photons during the exposure.
(iii) Calculate the standard deviation as a percentage of the average number of photons during the exposure.

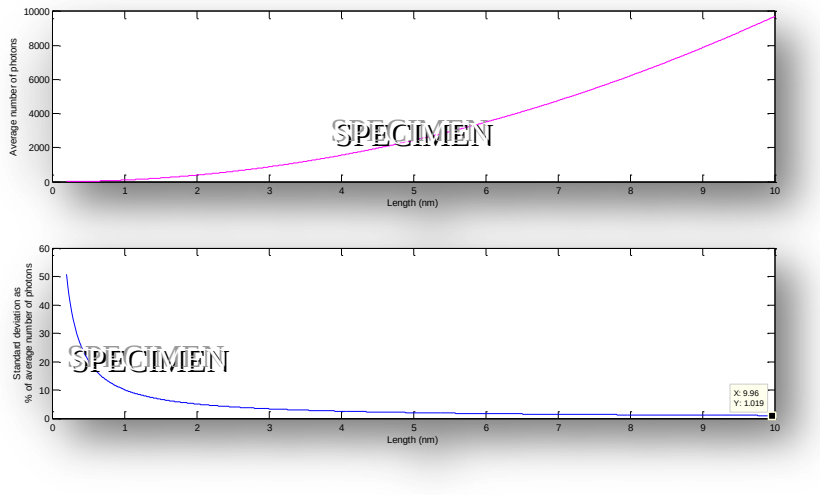
For an area of 10 nm x 10 nm.
(iv) Calculate the mean number of photons during the exposure.

(v) Calculate the standard deviation as a percentage of the average number of photons during the exposure.
(vi) Comment on the results.

Plot, as a function of length scale = $\sqrt{Area}$:

(vii) The mean number of photons during the exposure.
(viii) The standard deviation, as a percentage of the average number of photons, during the exposure.

You should get something like this.



(ix) If the maximum standard deviation, as a % of the average allowable for the process, is 5%, what is the minimum area that a resist feature can occupy on the wafer?
(x) What is the mean number of photons contributing to the Dose for the area calculated for part (ix)?

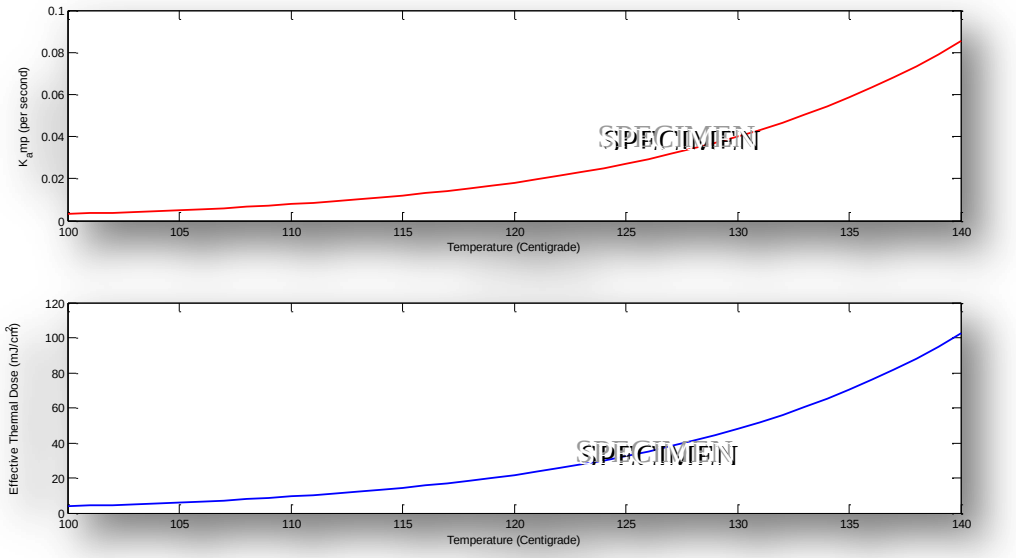
4. Plot the Amplification Rate Constant and Effective Thermal Dose versus Temperature for a resist with an amplification reaction E_a = 25.0 kcal/mol and A_r = exp(28.0) s^{-1}, and an exposure rate constant C = 0.05 mJ/cm^2. Use a temperature range from 100 to 140 ºC and assume a PEB time of 60 s. Use the following equations:

$$K_{amp} = A_r e^{-\frac{E_a}{RT}} \qquad (6.10)$$

and

$$Effective\ Thermal\ Dose = K_{amp} t_{PEB}/C$$

You should get something like this.



From your graphs, what are the values of the Amplification Rate Constant and the Effective Thermal Dose at 125 ºC?

5. The variance in acid concentration following exposure is given by the following Equation:

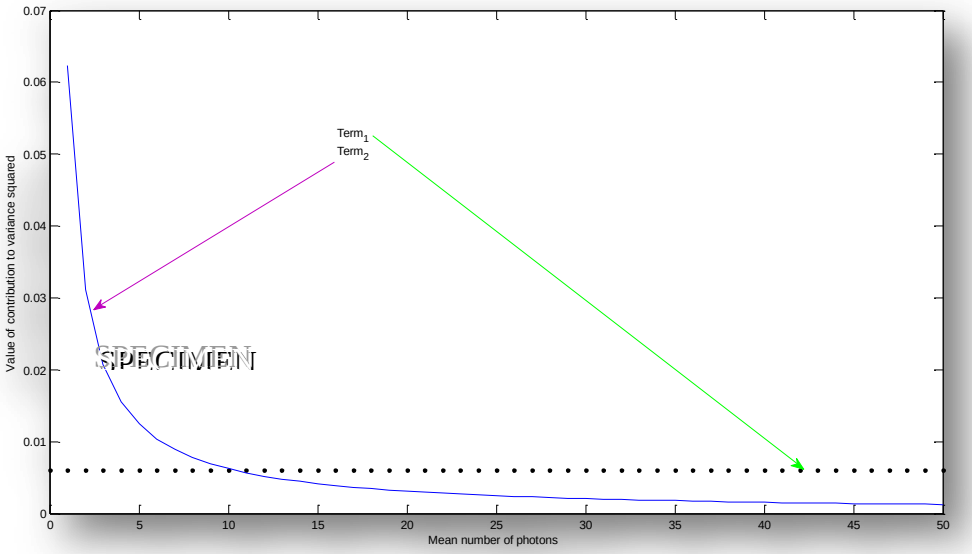
$$\sigma_h^2 = \frac{\langle h \rangle}{\langle n_{0-PAG} \rangle} + \frac{[(1-\langle h \rangle)\ln(1-\langle h \rangle)]^2}{\langle n \rangle} \qquad (6.70)$$

The first term on the right hand side of the Equation is the Poisson result based on exposure kinetics – the relative uncertainty in the resulting acid concentration after exposure is proportional to the reciprocal of the square root of the mean number of acid molecules generated in the volume of interest. For large volumes and exposure doses, the number of acid molecules generated is large and the statistical uncertainty becomes small. The second term accounts for photon shot noise.

Assume that a certain amount of acid is required to achieve a desired lithographic effect, that is, assume the mean concentration of photogenerated acid is fixed.

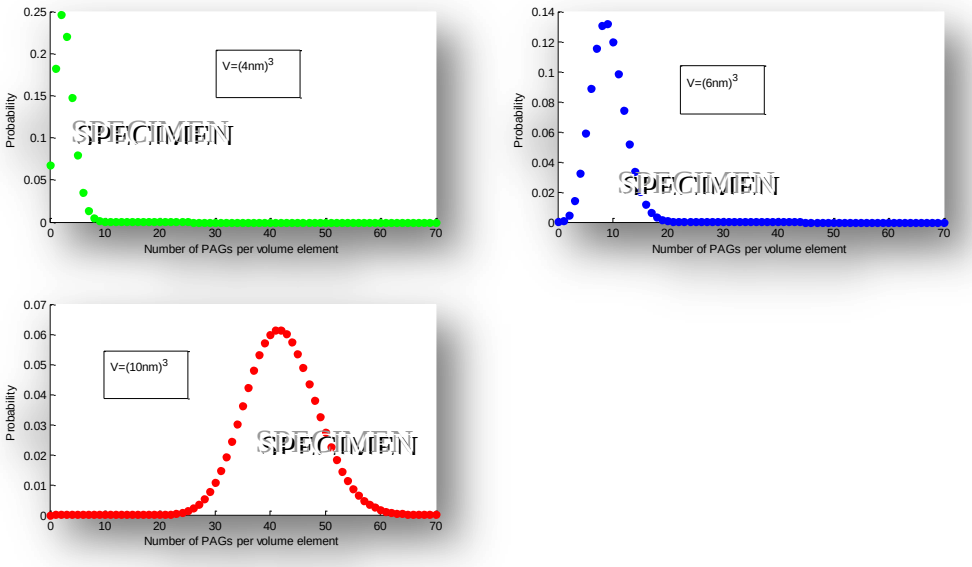
Plot both terms as a function of the mean number of photons. By noting the crossover point, identify the point at which the photon shot noise exceeds the PAG loading shot noise. Take the normalised acid concentration to be $\langle h \rangle$ = 0.3 and $\langle n_{0-PAG} \rangle / V$ = 0.05 /nm^3. Perform the calculation for a cube 10 nm on a side.

You should get something like this.



6. Consider a typical 193 nm resist. Take $G_0N_A = 0.042/nm^3$, so there are 0.042 molecules of PAG per cubic nm. Plot the Poisson distribution of the number of PAGs per volume element. Do the plot for cubic sample volumes, 4 X 4 X 4 nm, 6 X 6 X 6 nm and 10 X 10 X 10 nm.

For the 10 X 10 X 10 nm volume, calculate the mean and show that it matches that read from the graph. Your plots should look like this.



7. Photoresist Development

1. Consider the 'Original Mack model' for development, as given by Eq (7.10).

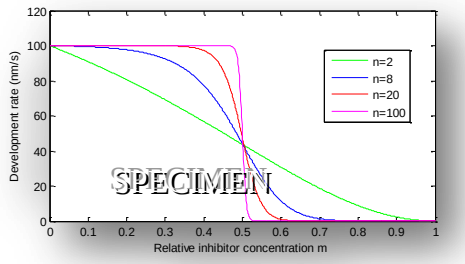
i) Plot the Development rate, r, of a resist as a function of the relative inhibitor concentration, m, for a few different values of n, the dissolution selectivity parameter. Take r_{max} = 100 nm/s, r_{min} = 0.1 nm/s. Use values of n = 2, 8 and 20. Take m_{th} = 0.5.

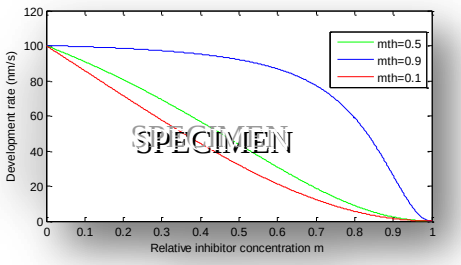
ii) Demonstrate that for high molecular weight resins, that is, for high values of n, the shape of the graph approaches a step function, giving an ideal threshold resist. What is the disadvantage of using these high molecular weight resins in resists?

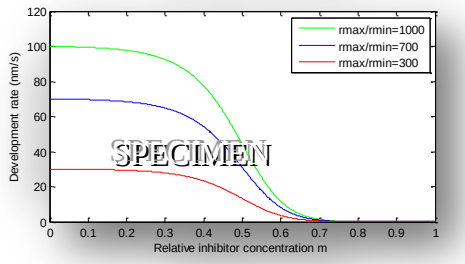
iii) Change the value of the threshold inhibitor concentration, m_{th}, and describe the effect on the graph.

iv) Change the value of r_{max}/r_{min} and show that increasing this ratio will increase lithographic performance.

You should get something like this.







2. Use the enhanced kinetic model, given by Eq (7.12), to plot Development Rate vs. Relative Inhibitor Concentration under the following conditions. Take r_{max} = 100 nm/s, r_{min} = 0.1 nm/s, r_{resin} = 10 nm/s. Take the rate constant for the enhancement reaction to be 9 and the rate constant for the inhibition mechanism to be 20.

i) Perform the plot for l = 9 and n = 1, 5, 9. Above what value of the relative inhibitor concentration do the n = 5 and n = 9 plots coincide?

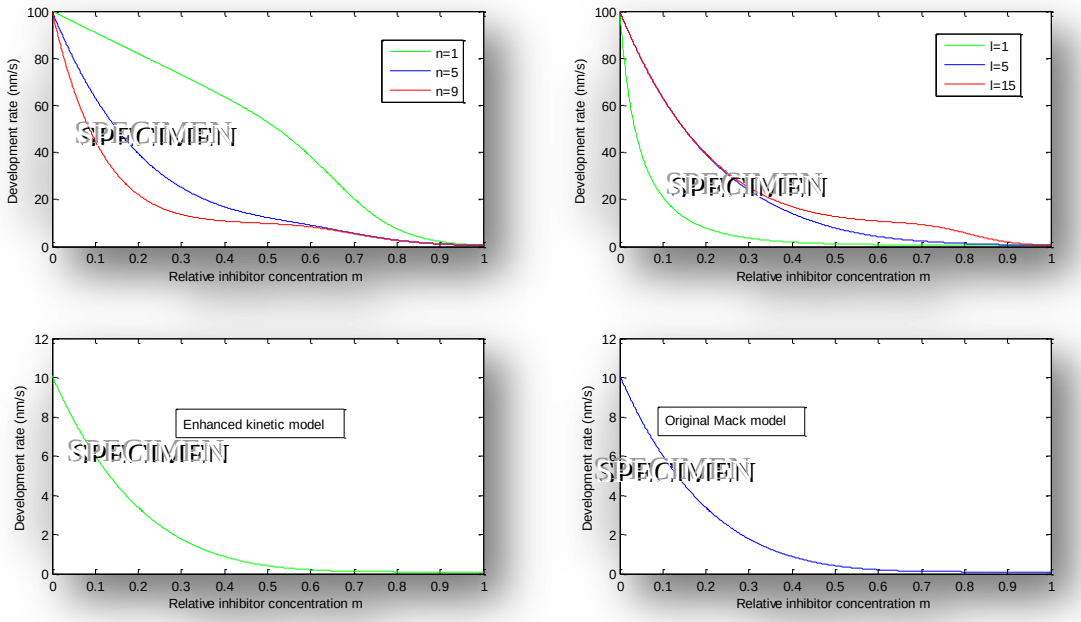
ii) Perform the plot for n = 5 and l = 1, 5, 15. Below what value of the relative inhibitor concentration do the l = 5 and l = 15 plots coincide?

iii) As noted in the text, the enhanced kinetic model for resist dissolution is a superset of the reaction-controlled version of the Original Mack model. The two models are the same when the parameter a >> 1. In addition, k_{inh} must be zero, that is, the inhibition mechanism should be unimportant and $r_{resin} = r_{min}$ and $r_{max} = r_{resin}\, k_{enh}$.

If the following parameter set is used, all these conditions are satisfied. By plotting both models, prove they give identical results.

r_{max} = 10 nm/s, r_{min} = 0.1 nm/s, r_{resin} = 0.1 nm/s, k_{enh} = 100, k_{inh} = 0, n = 5, l = 15.

You should get plots similar to this.



3. Consider a conventional positive resist that obeys the reaction-controlled version of the original kinetic development model of the equation:

$$r = r_{max}(1-m)^n + r_{min} \qquad (7.11)$$

Assume no bleaching occurs, but absorption means that the inhibitor concentration *m* can be related to incident dose *E* and depth into resist *z* by

$$m = e^{-CEI_r(z)} \quad with\ I_r(z) = e^{-\alpha z}$$

Using MATLAB®, plot Development Rate versus Dose up to 100 mJ/cm^2 for r_{max} = 200 nm/s, r_{min} = 0.05 nm/s, n = 4, C = 0.02 cm^2/mJ, α = 0.5 μm^{-1} and a depth into the resist of 500 nm. Compare these results to the approximate equation, which is valid for low doses only.

$$r(z) \approx r_{max}\left(1-m(z)\right)^n \approx r(z=0)e^{-\alpha n z} \qquad (7.19)$$

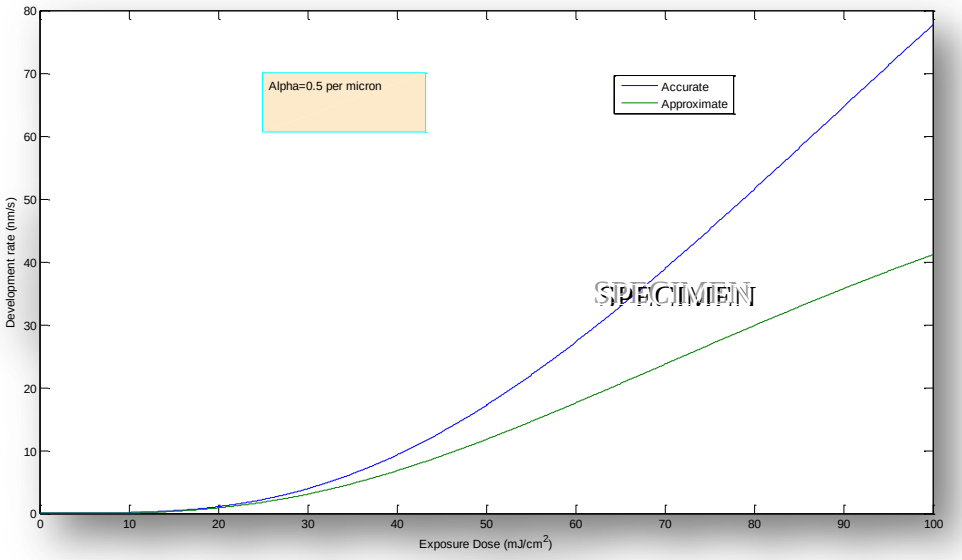
Repeat for α = 0.1 μm^{-1} and α = 1.5 μm^{-1}.

Answer the following questions.

i) Is the assumption, made in the derivation of the approximate equation, that we assume small exposure doses, apparent in your simulations?

ii) Based on your results, would the approximate equation be useful if the resist was highly absorbing?

The result for α = 0.5 μm^{-1} is shown below.



4. A conventional positive resist obeys the reaction-controlled version of the original kinetic development model of the equation viz.

$$r = r_{max}(1-m)^n + r_{min} \qquad (7.11)$$

If we assume that no bleaching occurs, the inhibitor concentration m can be related to dose E by

$$m = e^{-CE}$$

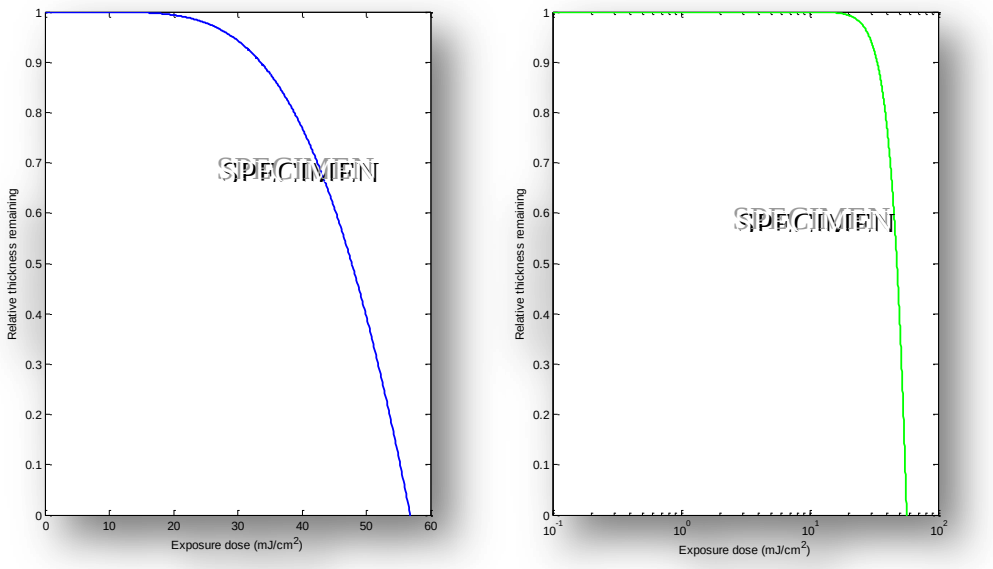
Assuming that the minimum development rate r_{min} can be ignored over the dose range of interest and ignoring any variation in development rate with depth into the resist, it can be shown that the relative thickness of the remaining resist is given by

$$T_r = 1 - \frac{r_{max} t_{dev}}{D}(1 - e^{-CE})^n$$

Take r_{max} = 200 nm/s, n = 7, C = 0.02 cm^2/mJ, an initial resist thickness of 800 nm and a development time of 60 s.

i) Calculate the Dose to Clear for this situation by letting $T_r = 0$ and solving for E_0.

ii) Plot the relative resist characteristic curve, T_r , versus (a) Exposure dose and (b) $\log_{10}$ (Exposure Dose). Mark the Dose-to-Clear on the plot. How does it compare to the calculated value?
Your plot should look like this.



5. Near the line edge, an aerial image can be approximated by a Gaussian. Consider an aerial image caused by 3-beam imaging of a dense pattern with pitch p expanded in a Taylor series about $x = 0$, the centre of the space.

$$I(x) = a_0 + a_1 cos\left(\frac{2\pi x}{p}\right) + a_2 cos\left(\frac{4\pi x}{p}\right)$$

$$= (a_0 + a_1 + a_2) - \left(\frac{a_1 + 4a_2}{2}\right)\left(\frac{2\pi x}{p}\right)^2 + \left(\frac{a_1 + 16a_2}{24}\right)\left(\frac{2\pi x}{p}\right)^4 - \cdots \quad (7.89)$$

For the Gaussian image,

$$I(x) = I_0 e^{\frac{-x^2}{2\sigma^2}} = I_0 - \left(\frac{I_0}{2\sigma^2}\right)x^2 + \left(\frac{I_0}{8\sigma^4}\right)x^4 - \cdots \quad (7.90)$$

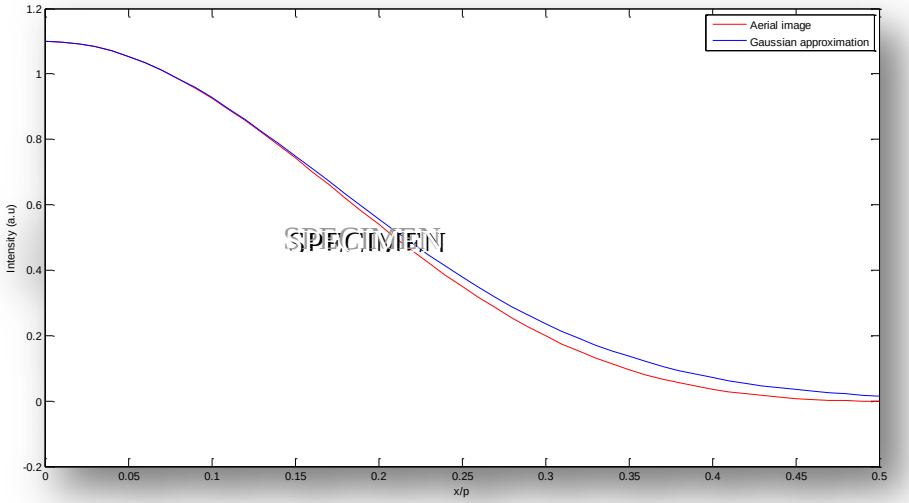
The zero- and second-order terms for these two series can be made the same as follows:

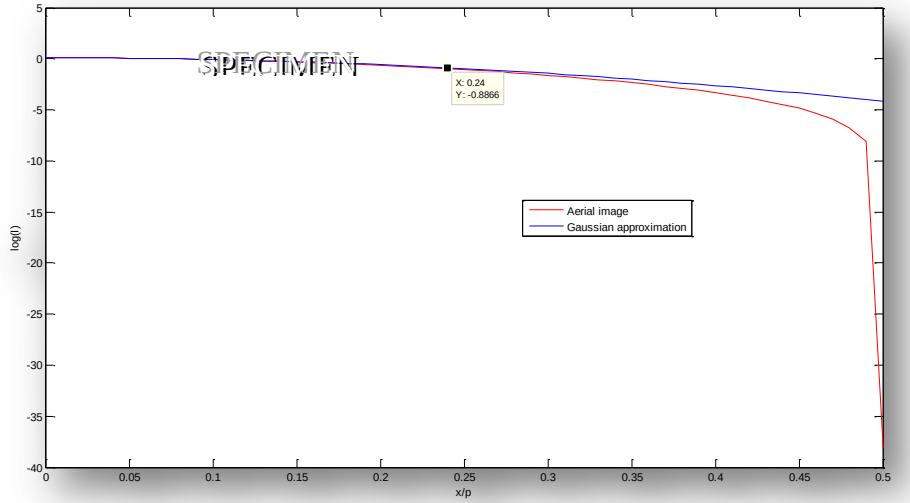
$$I_0 = (a_0 + a_1 + a_2) \quad \text{and} \quad \sigma^2 = \left(\frac{a_0 + a_1 + a_2}{a_1 + 4a_2}\right)\left(\frac{p}{2\pi}\right)^2 \quad (7.91)$$

i) Plot the aerial image, as given in Eq (7.89), and the Gaussian approximation to the aerial image, as given in Eq (7.90), on the same plot. Take $a_0 = 0.45$, $a_1 = 0.55$, $a_2 = 0.1$. Calculate I_0 and σ^2 from Eq (7.91). Do the plot for $x/p = 0$ to 0.5 on a linear-linear scale, with $p = 1$.

ii) Redo the plot on a log-linear scale. Measure the Image Log-Slope at the line edge by measuring the slope of the Gaussian at the line edge in this plot. Compare your answer to the value of x/σ^2 given by Eq (7.92).

Your plots should look something like this.


Aerial image
Gaussian approximation
Intensity (a.u)
x/p
SPECIMEN


Aerial image
Gaussian approximation
log(I)
x/p
X: 0.24
Y: -0.8866
SPECIMEN

8. Lithographic Control in Semiconductor Manufacturing

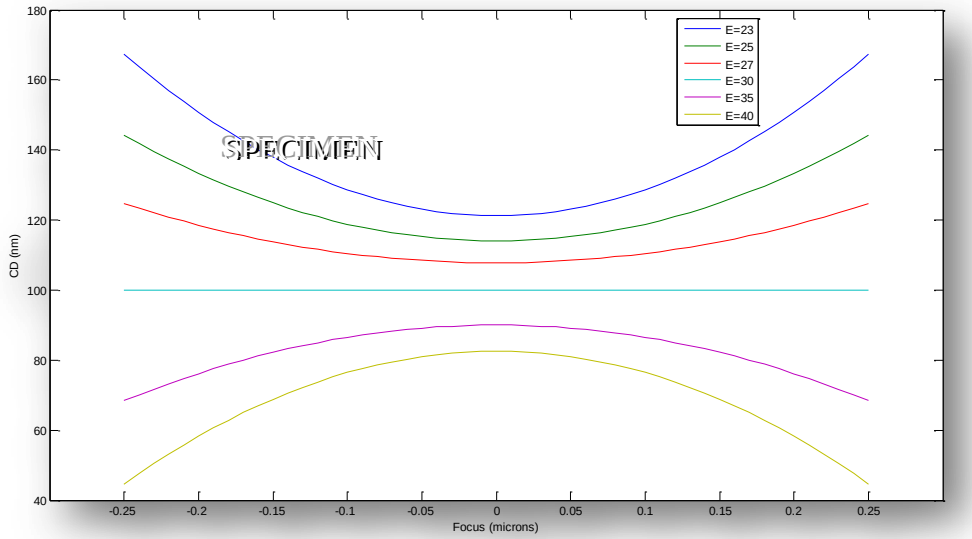
1. Use the Equations below to generate a theoretical Bossung plot for the following conditions.

$$\left.\frac{d\ ln\ CD}{d\ln E}\right|_{E_1,F_0} = -0.7, \quad \Delta = 0.17\ \mu m, \quad CD_1 = 100\ nm, \quad E_1 = 30\ mJ/cm^2$$

$$\frac{\mathrm{CD}-\mathrm{CD}_1}{\mathrm{CD}_1} = \left.\frac{d\ ln\ CD}{d\ ln\ E}\right|_{E=E_1}\left(1-\frac{\mathrm{E}_1}{E}\right) \qquad (8.31)$$

$$\frac{d\ ln\ CD}{d\ ln\ E} = \left.\frac{d\ ln\ CD}{d\ ln\ E}\right|_{F=F_0}\left[1+\left(\frac{F-F_0}{\Delta}\right)^2\right] \qquad (8.37)$$

Do the plot for doses of 23, 25, 27, 30, 35 and 40 mJ/cm^2. You should get something like this.



Answer the following questions:

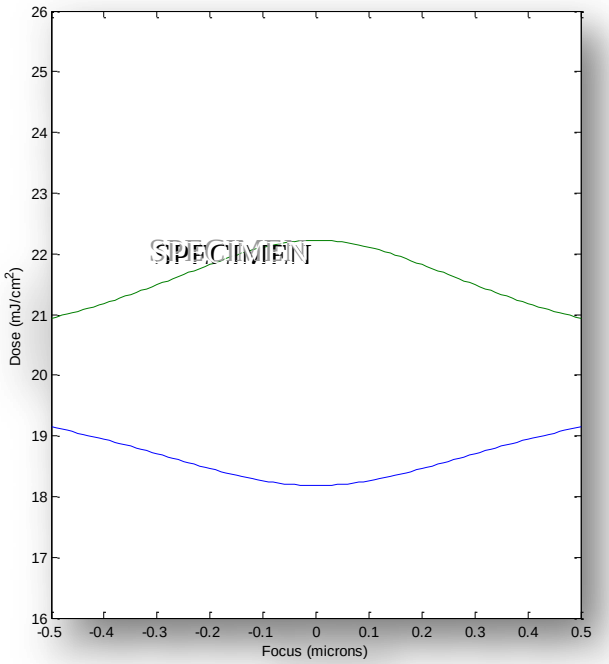
i) At what value of dose is the Bossung plot flat? Comment on this.

ii) What is the effect on the plot of increasing Δ? Comment on this.

iii) What is the effect on the plot of increasing CD_1? Comment on this.

iv) What is the effect on the plot of decreasing E_1? Comment on this.

2. Use Eq (8.49) to plot the process windows for a nominal line *CD* of 100±10 nm. The nominal dose is 20 mJ/cm^2, s = -1, Δ = 0.45 microns. Isofocal CD_1 = 100 nm. You should get something like this.



3. Equations (8.31) and (8.35) both predict CD versus dose, with the latter equation being more accurate.

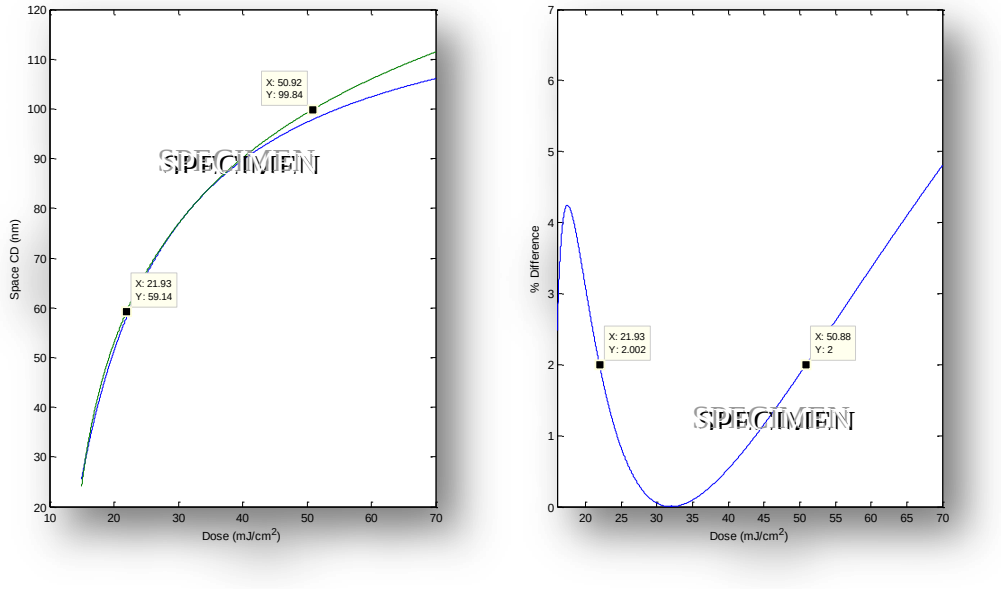
$$\frac{\mathrm{CD}-\mathrm{CD}_1}{\mathrm{CD}_1} = \left.\frac{d\ ln\ CD}{d\ ln\ E}\right|_{E=E_1} \left(1 - \frac{\mathrm{E}_1}{E}\right) \qquad (8.31)$$

$$\frac{CD}{CD_1} = \sqrt{1 + 2\left.\frac{dlnCD}{dlnE}\right|_{E=E_1} ln\left(\frac{E}{E_1}\right)} \qquad (8.35)$$

Plot CD versus Dose for these two models on one graph for the case of CD_1 = 80 nm, E_1 = 32 mJ/cm^2 and $\left.\frac{d\ ln\ CD}{d\ ln\ E}\right|_{E=E_1} = 0.6$.

Plot the percentage difference in CD predicted by the two models as a function of Dose. At what Dose values do the two models disagree with each other in predicted *CD* by 2%?

Here are some typical plots



4. Using the Equations below, (8.50) – (8.52), plot the depth of focus of a 65 nm line as a function of isofocal bias. Use the following values: CD_{spec} = 0.2, El_{spec} = 0.08, s_{nom} = 0.75, and Δ = 0.2 μm.

For isofocal bias within the CD spec,

$$DOF = 2\Delta\sqrt{\frac{CD_{spec}}{|s_{nom}|EL_{spec}} - 1} \qquad (8.50)$$

For isofocal bias greater than the CD spec range,

$$DOF \approx \frac{1}{K}2\Delta\sqrt{\frac{CD_{spec}}{|s_{nom}|EL_{spec}} - 1} \qquad (8.51)$$

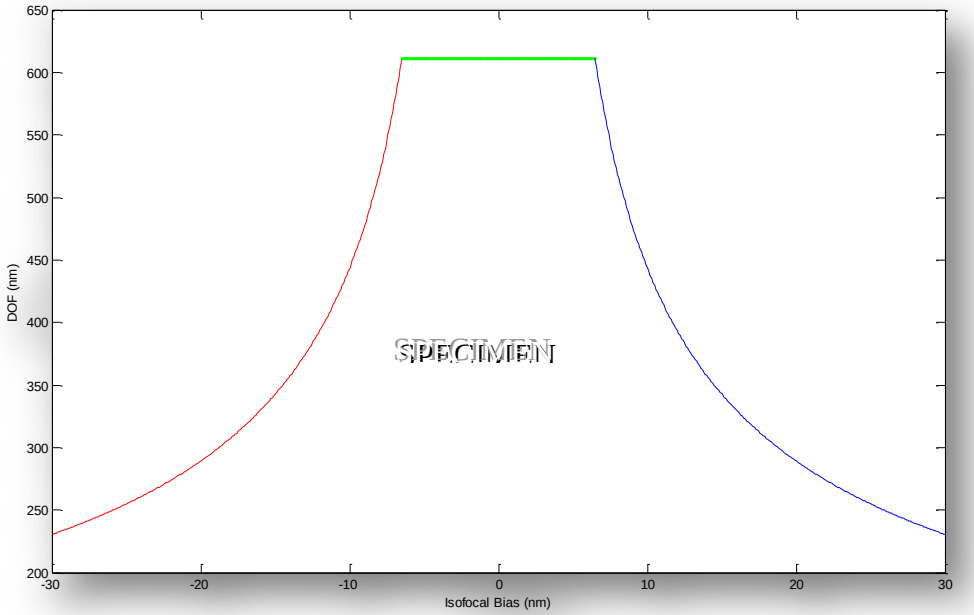
where

$$K = \sqrt{\frac{CD_{rel-iso-limit}}{|s_{nom}|EL_{spec}} + 1}$$

and

$$CD_{rel-iso-limit} = \frac{|CD_1 - CD_{limit}|}{CD_{nom}} \qquad (8.52)$$

You should get something that looks like this.


650
600
550
500
450
400
350
300
250
200
DOF (nm)
-30
-20
-10
0
10
20
30
Isofocal Bias (nm)
SPECIMEN

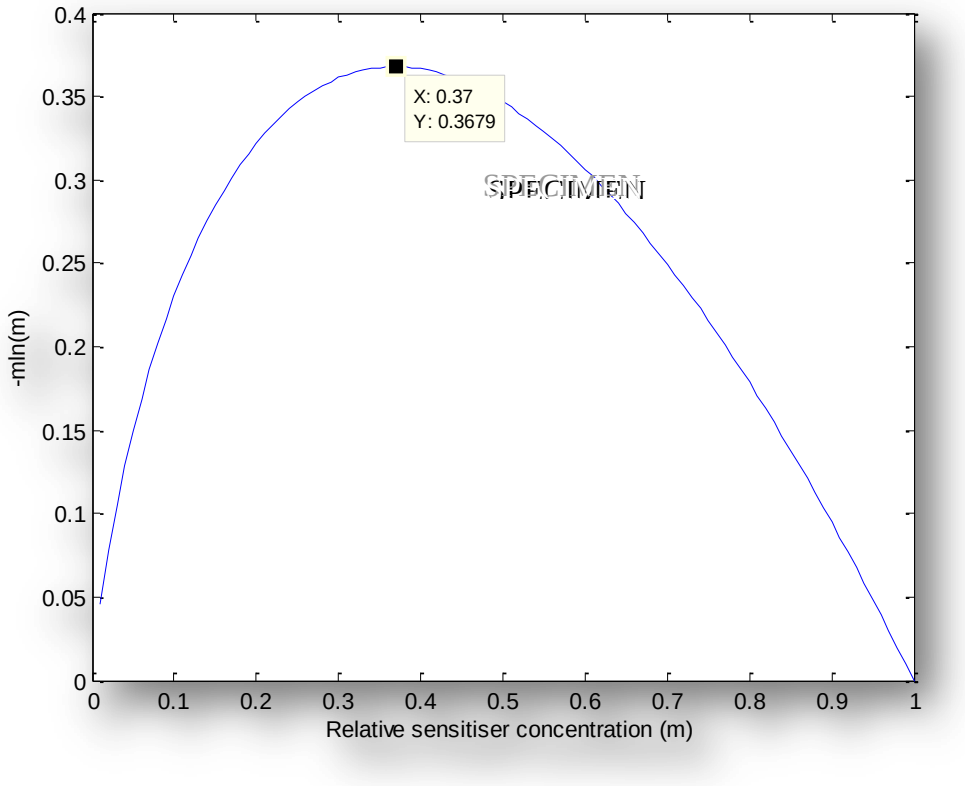
9. Gradient-Based Lithographic Optimization: Using the Normalized Image Log-Slope

1. The slope of the latent image is given by

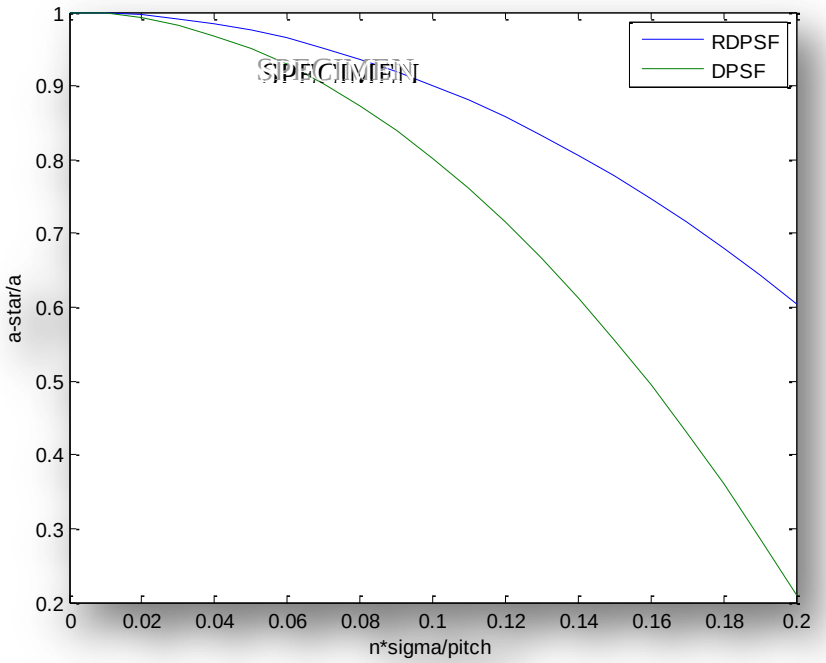
$$\frac{\partial m}{\partial x} = mln(m)\frac{\partial lnI}{\partial x} \qquad (9.11)$$

The quantity *mln*(m), relates the Image Log Slope to the latent image gradient. Plot the value of -*mln*(m) against *m*, the relative concentration of light sensitive material, for values of m between 0 and 1.

Using your plot, show that for $m = 0$ or $m = 1$, $mln(m) = 0$. At these values the gradient of the latent image is zero and no information is transferred to the wafer. This makes sense since at $m = 0$ the dose is infinite, while for $m = 1$, the dose is zero.
Show also that at $m = 0.37$ the value of *mln(m)* is at a maximum value of around 0.37 and the full information contained in the aerial image is transferred to the wafer.
Your plot should look something like this.



2. Plot $\frac{a_n^*}{a_n}$ against $n\sigma_D/pitch$ to compare the degradation of the Fourier coefficients as a function of increasing $n\sigma_D/pitch$ for the diffusion only case and the reaction-diffusion system. Use the Taylor Series expansions of the exponential given in Eq (9.35). You should get something like this.



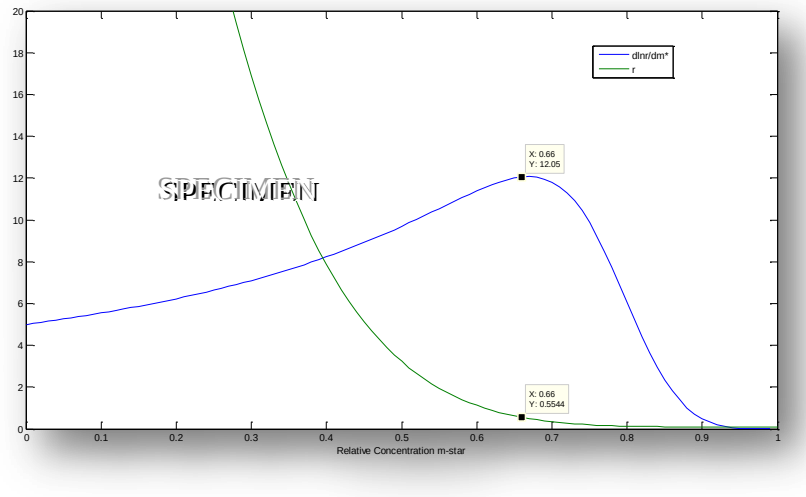
3. Using the non-linear development rate function

$$r = r_{max}\,(1 - m^*)^n + r_{min} \qquad (9.55)$$

it can be shown that

$$\frac{\partial lnr}{\partial m^*} = \frac{n}{1-m^*}\left(1 - \frac{r_{min}}{r}\right) \qquad (9.56)$$

i) Plot $\frac{\partial lnr}{\partial m^*}$ and r as function of m^*. Take n = 5, r_{max} = 100 nm/s, r_{min} = 0.1 nm/s. You should get something like this.



ii) Show that the maximum gradient occurs ($-\frac{\partial lnr}{\partial m^*}$ is a maximum) when $r = n\,r_{min}$.

iii) Show that this maximum gradient occurs at a value of $m^* = 1 - \left[\frac{(n-1)r_{min}}{r_{max}}\right]^{\frac{1}{n}}$.

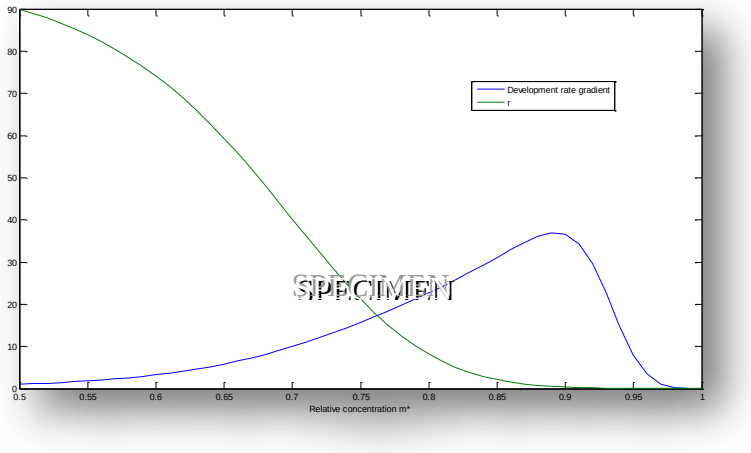
4. Use the Full Kinetic Rate model (the *Mack* model):

$$r = r_{max}\frac{(a+1)(1-m^*)^n}{a+(1-m^*)^n} + r_{min} \qquad (9.60)$$

$$a = \frac{(n+1)}{(n-1)}(1 - m_{th})^n \qquad (7.9)$$

$$\frac{\partial lnr}{\partial m^*} = \frac{n}{1-m^*}\left(1 - \frac{r_{min}}{r}\right)\left(\frac{a}{a+(1-m^*)^n}\right) \qquad (9.61)$$

i) Plot r and $\frac{\partial lnr}{\partial m^*}$ as a function of m^* on the same plot. Take n = 5, r_{max} = 100 nm/s, r_{min} = 0.1 nm/s, m_{th} = 0.7. You should get something like this.



ii) What inhibitor concentration maximizes the development rate gradient?

iii) Show that the maximum development rate gradient occurs at a value of r given by

$$r = \frac{nr_{min}}{1+(n-1)\frac{(1-m^*)^n}{(1-m_{th})^n}} \qquad (9.62)$$

iv) Show, with a suitable plot, that for large n, say n = 25, and since the calculation is done for a good resist (r_{max}/r_{min} >1000), the development rate can be approximated as

$$\frac{-\partial lnr}{\partial m^*}\Big|_{max} \approx \frac{n}{(1-m_{th})} + \frac{1}{2(1-m_{th})} ln\left(\frac{r_{max}}{r_{min}}\right) \qquad (9.66)$$

v) Show, with a suitable plot, that for large n, say for n = 25, the optimum inhibitor concentration approaches m_{th} (which can be taken to be 0.7).

10. Resolution Enhancement Technologies

1. From the book, the electric field intensity of the ideal aerial image of small lines and spaces, using coherent illumination, for an ALT-PSM is given by

$$E(x) = a_0 + a_1 \cos\left(\frac{\pi x}{p}\right) \qquad (10.30)$$

with $I = |E|^2$ for imaging in air.

For an ideal ALT-PSM with phase error (see text),

$$a_0 \approx -i\Delta\emptyset\left(\frac{w_s}{2p}\right)$$

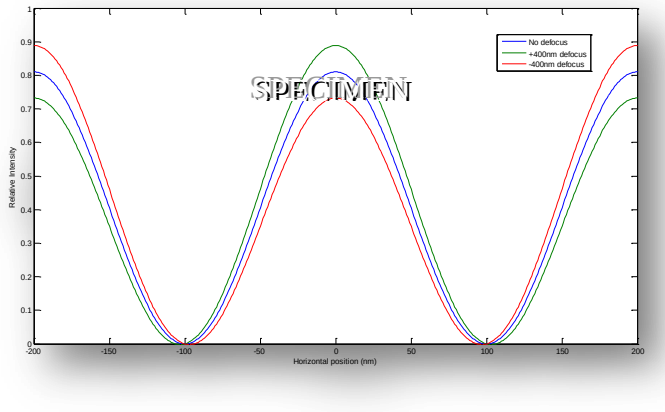
$$a_1 \approx \frac{2\sin\left(\frac{\pi w_s}{2p}\right)}{\pi}\left(1 + \frac{i\Delta\emptyset}{2}\right) \qquad (10.31)$$

If the image is defocused,

$$\mathrm{I}_{\mathrm{out\_of\_focus}} = \mathrm{I}_{\mathrm{in\_focus}}(\mathrm{x}) + 2\Delta\emptyset\left(\frac{w_s}{2p}\right) E_{ideal}(x)\sin\left(\pi\delta\lambda/p^2\right) \quad (10.33)$$

Plot the aerial image for an ALT PSM with a 10 degree phase error for
i) +400 nm defocus.
ii) An in focus image.
iii) -400 nm defocus.

Assume a dense line space pattern with 100 nm lines and spaces and, therefore, a pitch of 200 nm. The illumination is coherent and at a wavelength of 248 nm. You should get something like this.

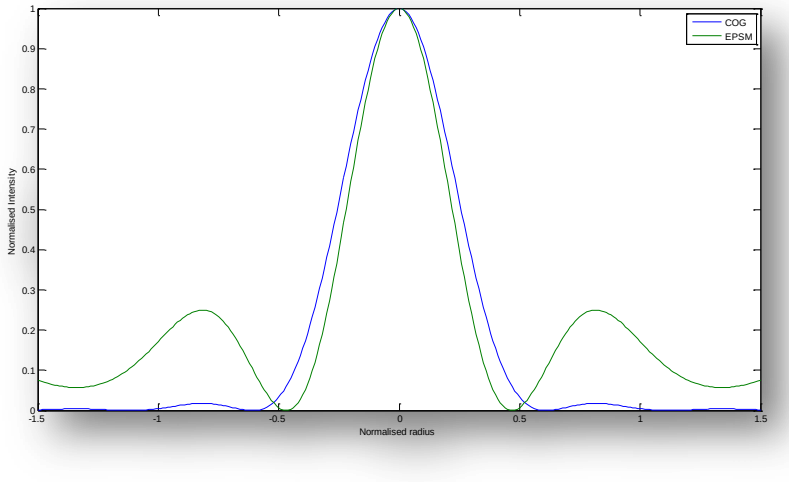


2. The point spread functions for a Chrome-On-Glass and Embedded PSM are given by the following equations.

$$PSF_{ideal\_COG} = \left|\frac{J_1(2\pi\rho)}{\pi\rho}\right|^2 \qquad (10.42)$$

$$PSF_{ideal\_EPSM} = \left|\frac{\frac{J_1(2\pi\rho)}{\pi\rho} - t}{1-t}\right|^2 \qquad (10.43)$$

Plot the PSFs for a COG and EPSM. The plot should be for normalized radii of -1.5 to +1.5 and you can assume the ESPM has an intensity transmittance of 6%. You should get something like this.



(i) Measure the FWHM of both PSFs and compare them.

(ii) Measure the sidelobe intensity in the EPSM.

(iii) Rerun the simulation at transmittances of 1 - 6% and measure the sidelobe intensity for the PSF of the EPSM. Plot the sidelobe intensity and the FWHM of the EPSM as a function of transmittance and comment on the graphs.

A MATLAB® Primer

Introduction

MATLAB® is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation. Using MATLAB®, you can solve technical computing problems faster than with traditional programming languages, such as C, C++ or Fortran.
The purpose of this Primer is to help you to begin to use MATLAB® so that you can do the problems in this Manual. Since most of the problems in this Manual require you to plot complex mathematical functions, this plotting capability will form the core of the Primer. MATLAB® is far more powerful and versatile than this Primer might suggest. In order to explore these features, please go to www.mathworks.com, where there are a wealth of tutorials, webinars, examples and other educational material available on the use of the software.

Plotting using MATLAB®

MATLAB® can produce 2D and 3D plots of curves and surfaces. In this Primer, we will use MATLAB® to plot the improvement in Depth of Focus, $\frac{DOF_{immersion}}{DOF_{dry}}$, for a binary mask pattern of lines and spaces as a function of pitch. This requires plotting Eq (3.83).

$$\frac{DOF_{immersion}}{DOF_{dry}} = \frac{1-\sqrt{1-\left(\frac{\lambda}{p}\right)^2}}{n-\sqrt{n^2-\left(\frac{\lambda}{p}\right)^2}} \qquad (3.83)$$

We will produce the plot for pitches between 200 nm and 600 nm. The illumination source has a wavelength of 193 nm. The die is immersed in ultrapure degassed water with a refractive index of 1.43.

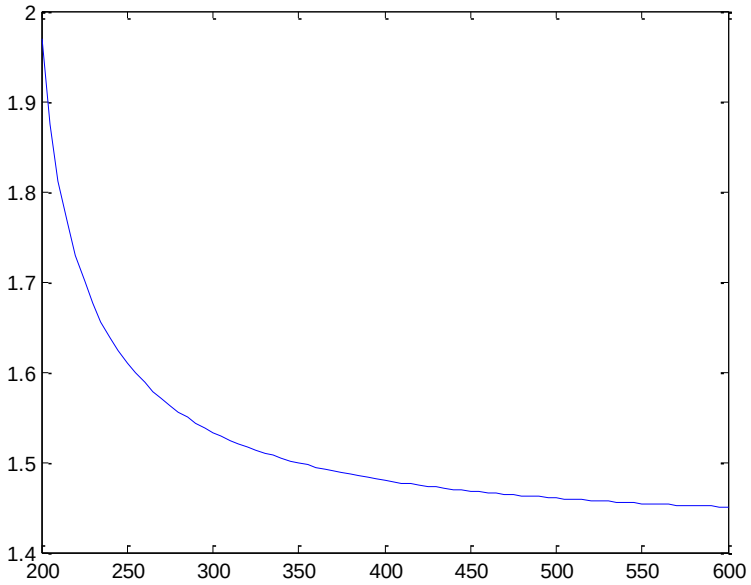
The code to perform the plot is shown below and illustrates most of the features of MATLAB® required to produce the plots in this Manual. As you can see, MATLAB® is a powerful language, and complex plots can be produced using only a few lines of code.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                                                      %
%                                                      %
%     Primer Solution :- Kevin Berwick                 %
%     Summer 2010                                      %
%                                                      %
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%  define constants


clear
pitch=200:5:600
lambda = 193
n=1.43
% Calculate the value of Eq 3.83

DOF_immersion_dry = (1-sqrt(1-(lambda./pitch).^2))./(n-sqrt(n^2-(lambda./pitch).^2))
% Plot image
plot(pitch, DOF_immersion_dry)
```

The script starts with some comments, which are ignored by MATLAB®. Comments are delineated by beginning them with the % sign, as shown. In the Editor, comments are given a green colour.
The first statement in the script is ***clear***. This removes all variables from the workspace, releasing them from system memory.
Next we define an array *pitch*, which has values from 200 to 600 with a step size of 5.
lambda, the wavelength and *n*, the refractive index, are constants and are defined as shown, with an equals sign. Note that MATLAB® is case sensitive; it distinguishes between uppercase and lowercase letters. *A* and *a* are not the same variable.
Next we calculate the value of the Equation, using a small selection of the software's vast number of built-in functions. Most of this line is self explanatory, however the use of the dot when performing operations involving the pitch array is worth exploring further. The expression

(lambda./pitch).^2

uses the dot twice, one to divide the value of *lambda*, a single value, by the *pitch*, which is an array. What happens is that *lambda* is divided by each element in the array *pitch.* The result of this element-wise division is a new array *(lambda./pitch)*. When we want to square this array, we again have to use the dot to show that each element in the array is to be squared. Leaving out the dot in either case would lead to an error.
We finish the script by using the keyword ***plot*** followed by the x and y values, in brackets, that we want to plot.
Running the script gives the following output. Both axes are automatically scaled and the default blue colour is used for the plot.



Now that we have the plot, we can improve its appearance using the plot tools provided by MATLAB®. We can add axis titles, legends and change the colours used in the plot with these tools. If desired, we can then generate an M-file so that we can ensure that the changes in the appearance of the final plot are permanently encoded into a MATLAB® script. In addition, we can perform basic fitting and even rotate the plot in 3D in order to assist in data visualisation.

However, these are merely details. Most of the exercises in this Manual can be done simply by building on the MATLAB® example given in this Primer.

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